4/18/2018

$$\boxed{A^\mu = \frac{e V^\mu}{V\cdot(x-r)}} \quad (14.6)$$

$$r^\mu(\tau) \qquad v^\mu = \frac{dr^\mu}{d\tau}$$

$x - r = \begin{pmatrix} c(t-t') \\ \vec{R} \end{pmatrix}$ on light cone. $\begin{pmatrix} R \\ \vec{R} \end{pmatrix}$

$$V^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix} \qquad V\cdot(x-r) = (\gamma c)(R) - (\gamma \vec{v}\cdot\vec{R})$$

$$= \gamma c R(1 - \vec{\beta}\cdot\hat{n})$$

$$\Phi(\vec{x},t) = \frac{e}{R(1-\hat{n}\cdot\vec{\beta})}\Bigg|_{t'_{ret}} \qquad \vec{A} = \frac{e\,\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})}\Bigg|_{t_{ret}}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$ ← differentiate inside integral

$$\partial^\mu A^\nu = 2e \int d\tau\, V^\nu(\tau)\, \Theta(x^0 - r^0(\tau))\, \partial^\mu \delta[(x-r(\tau))^2]$$

$$\partial^\mu \delta[f] = \frac{\partial}{\partial f}\delta[f]\,\partial^\mu f = \partial^\mu f \cdot \frac{d(\delta[f])/d\tau}{df/d\tau}$$

2

$$f = (x - r(\tau))^2 \qquad \partial^\mu f = 2(x-r)^\mu$$

$$\frac{df}{d\tau} = 2(x-r)\cdot\left(-\frac{dr}{d\tau}\right) = -2V\cdot(x-r)$$

$$\partial^\mu A^\nu = 2e\int d\tau\, \frac{d}{d\tau}\left[\frac{(x-r)^\mu V^\nu}{V\cdot(x-r)}\right]\theta(x^0-r^0)\,\delta[(x-r)^2]$$

↑ parts

$$F^{\mu\nu} = \frac{e}{V\cdot(x-r)}\frac{d}{d\tau}\left[\frac{(x-r)^\mu V^\nu - (x-r)^\nu V^\mu}{V\cdot(x-r)}\right]$$

$$(x-r)^\mu = R\begin{pmatrix}1\\ \hat{n}\end{pmatrix} \qquad V^\mu = \gamma c\begin{pmatrix}1\\ \vec{\beta}\end{pmatrix}$$

$$\frac{dV^\mu}{d\tau} = A^\mu = \begin{pmatrix} c\gamma^4\, \vec{\beta}\cdot\dot{\vec{\beta}} \\ c\gamma^2\dot{\vec{\beta}} + c\gamma^4\,\vec{\beta}(\vec{\beta}\cdot\dot{\vec{\beta}})\end{pmatrix}$$

$$\frac{d}{d\tau}\left(V\cdot(x-r)\right) = -\underbrace{V\cdot V}_{-c^2} + (x-r)\cdot\frac{dV}{d\tau}$$

$$\gamma\frac{d\gamma}{dt} = \gamma\frac{d}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} = -\tfrac{1}{2}\cdot\gamma\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2\dot{v}v}{c^2}\right) = \gamma^4 v\dot{v} = \gamma^4\,\vec{\beta}\cdot\dot{\vec{\beta}}$$

③

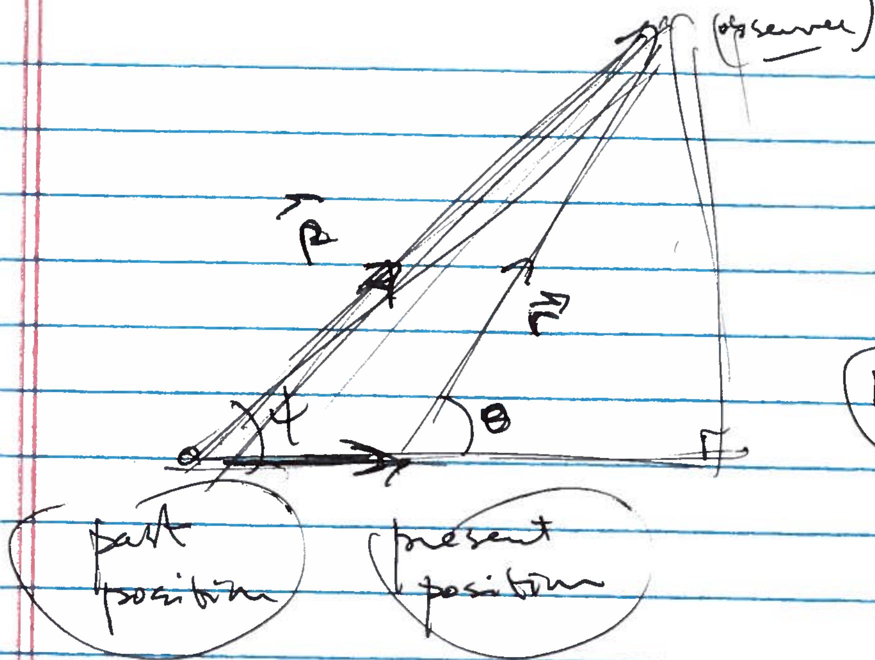
$$\vec{E} = \frac{e(\hat{n}-\vec{\beta})}{\gamma^2(1-\hat{n}\cdot\vec{\beta})^3}\frac{1}{R^2} + \frac{e}{c}\frac{\hat{n}\times\left[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{(1-\hat{n}\cdot\vec{\beta})^3\cdot R}\Bigg|_{t'_r}$$

$$\vec{B} = \hat{n}\times\vec{E} \qquad (14.14)$$

R^{-2} independent of $\dot{\vec{\beta}}$ $\Longrightarrow$ Boosted Coulomb field

R^{-1}. $\dot{\vec{\beta}}$. radiation from accelerated q.

(4)



$R = c\Delta t$

$$\vec{v}\,\Delta t = \left(\frac{\vec{v}}{c}\right)(c\Delta t) = \vec{\beta} R$$

$$\vec{\beta} R + \vec{r} = \vec{R} \qquad \boxed{\vec{r} = R(\hat{n} - \vec{\beta})}$$

$$\vec{E} = \frac{eR(\hat{n} - \vec{\beta})}{\gamma^2 [R(1 - \hat{n}\cdot\vec{\beta})]^3}$$

$$[R(1 - \hat{n}\cdot\vec{\beta})]^2 = R^2 \cdot [1 - 2\hat{n}\cdot\vec{\beta} + (\hat{n}\cdot\vec{\beta})^2]$$

$$|\vec{r}|^2 = R^2 (1 - 2\hat{n}\cdot\vec{\beta} + |\vec{\beta}|^2)$$

$$= R^2 \left((1 - \hat{n}\cdot\vec{\beta})^2 + \underbrace{|\vec{\beta}|^2 - |\hat{n}\cdot\vec{\beta}|^2}\right)$$

$$\beta^2 \cdot R^2 \cdot \sin^2\psi = \beta^2 \cdot r^2 \sin^2\theta$$

$$\boxed{\vec{E} = \frac{e\vec{r}}{\gamma^2 r^3 (1 - \beta^2 \sin^2\theta)^{3/2}}}$$ as before (1).

radiation:

$$\vec{E} = \frac{e}{c}\,\frac{\hat{n}\times\left[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{R\,(1-\hat{n}\cdot\vec{\beta})^3}\Bigg|_{t_{ret}}$$

nonrelativistic: $|\vec{\beta}| \ll 1$,

$$\boxed{\vec{E} = \frac{e}{c}\,\frac{\hat{r}\times(\hat{r}\times\dot{\vec{\beta}})}{r}} \qquad \vec{B} = \hat{r}\times\vec{E}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0 c}\,\vec{E}\times(c\vec{B}) = \frac{c}{4\pi}\,\vec{E}\times\vec{B}$$

(14.19)

$$= \frac{c}{4\pi}\,\vec{E}\times(\hat{r}\times\vec{E})$$

$$\hat{r}(\vec{E}\cdot\vec{E}) - \vec{E}\,\cancel{(\hat{r}\cdot\vec{E})}$$

$$\boxed{\vec{S} = \frac{c}{4\pi}\,|\vec{E}|^2\,\hat{r}}$$

$$\frac{dP}{d\Omega} = r^2\,\hat{r}\cdot\vec{S} = \frac{c}{4\pi}\,\frac{e^2}{c^2}\,\left|\hat{r}\times(\hat{r}\times\dot{\vec{\beta}})\right|^2$$

$$\boxed{\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\,\left|\hat{r}\times(\hat{r}\times\dot{\vec{\beta}})\right|^2} \qquad (14.20)$$

(***)

(8)

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\left|\hat{r}\times(\hat{r}\times\ddot{\vec{x}})\right|^2 = \frac{e^2\dot{v}^2}{4\pi c^3}\sin^2\theta$$

$$\boxed{P = \frac{8\pi}{3}\frac{1}{4\pi c^3}e^2\dot{v}^2 = \frac{2}{3}\frac{e^2}{c^3}|\dot{v}|^2}$$

(14.22)

Instantaneous

Harmonic $\vec{x}\sim e^{-i\omega t}$ $\quad \vec{v} = \dot{\vec{x}} = -i\omega\vec{x}$ $\quad \dot{\vec{v}} = -\omega^2\vec{x}$

$$e\vec{x} = \vec{p}$$

$$\boxed{P = \frac{2}{3}\frac{\omega^4}{c^3}|\vec{p}|^2} \rightarrow \frac{2}{3}\frac{ck^4}{4\pi\varepsilon_0}|\vec{p}|^2$$

$4\pi\varepsilon_0$

$$\cancel{\langle P\rangle = \frac{1}{12\pi\varepsilon_0}ck^4|\vec{p}|^2}$$

$$\langle P\rangle = \frac{ck^4}{12\pi\varepsilon_0}|\vec{p}|^2$$

amplitude

$$\frac{1}{\varepsilon_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = cZ_0$$

Relativistic generalization

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\vec{p}}{dt}\right)\cdot\left(\frac{d\vec{p}}{dt}\right) \quad \text{(3-vector product)}$$

$$\longrightarrow \quad P = -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp}{d\tau}\right)\cdot\left(\frac{dp}{d\tau}\right) \quad \text{(4-vector product)}$$

Two ways to rewrite:

$$\frac{dE}{d\tau} = \gamma\frac{dE}{dt} = \gamma(\vec{F}\cdot\vec{v}) = \gamma\left(\frac{d\vec{p}}{dt}\right)\cdot\vec{v} = \vec{v}\cdot\frac{d\vec{p}}{d\tau}$$

$$E^2 = (mc^2)^2 + (c\vec{p})^2 \qquad E\frac{dE}{d\tau} = c^2\vec{p}\cdot\frac{d\vec{p}}{d\tau}$$

$$\hookrightarrow \quad \frac{dE}{d\tau} = c^2\frac{\vec{p}}{E}\cdot\frac{d\vec{p}}{d\tau} = c^2\frac{(\gamma m\vec{v})}{(\gamma mc^2)}\cdot\frac{d\vec{p}}{d\tau} = \vec{v}\cdot\frac{d\vec{p}}{d\tau}.$$

$$\frac{d\vec{p}}{d\tau} = \frac{d}{d\tau}(\gamma m\vec{v}) = \gamma\frac{d}{dt}(\gamma m\vec{v}) = \gamma^2 m\dot{\vec{v}} + \gamma m\vec{v}\,\dot{\gamma}$$

$$= \gamma^2 mc\dot{\vec{\beta}} + \gamma mc\vec{\beta}\,(\gamma^3\vec{\beta}\cdot\dot{\vec{\beta}})$$

$$\boxed{P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left((\dot{\vec{\beta}})^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right)}$$

Liénard 1898. Lorentz scalar!