4/20/2018

Accelerated charges.

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \left| \hat{r} \times (\hat{r} \times \dot{\vec{v}}) \right|^2$$

$$P = \frac{2}{3}\frac{e^2}{c^3} |\dot{v}|^2$$

$\to$ $P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\vec{p}}{dt}\right)\cdot\left(\frac{d\vec{p}}{dt}\right)$

$\to$ $P = -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp^\mu}{d\tau}\right)\left(\frac{dp_\mu}{d\tau}\right).$

scalar. $$P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left[\dot{\beta}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right]$$

Q: what is this quantity?

Frame invariant $\to$ Power in rest frame.

$d\vec{p} = 0$ $\quad dE' = \gamma(dE + \cancel{\vec{\beta}\cdot d\vec{p}}) = \gamma\, dE.$

$dt' = \gamma(dt + \cancel{\frac{\vec{\beta}\cdot d\vec{x}}{c^2}}) = \gamma\, dt.$

$$\left|\frac{dE'}{dt'} = \frac{dE}{dt}\right|$$ ~~what~~ ..

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<u>Case</u> $\vec{\beta} \parallel \dot{\vec{\beta}}$ $\qquad \underline{\vec{\beta} \times \dot{\vec{\beta}} = 0}$ $\qquad \gamma^6 \dot{\beta}^2$

linear accelerator. SLAC, ILC, ..

$$\frac{d\vec{p}}{dt} = \gamma m \dot{\vec{v}} + \cancel{\gamma} m \vec{v} \left(\gamma^3 \vec{\beta} \cdot \dot{\vec{\beta}} \right)$$

$$= \gamma m \dot{\vec{v}} \left(1 + \beta^2 \gamma^2 \right) = \gamma^3 m \dot{\vec{v}}$$

$$1 + \frac{\beta^2}{1-\beta^2} = \frac{1 - \cancel{\beta^2} + \cancel{\beta^2}}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2$$

$$\boxed{P = \frac{2}{3} \frac{e^2}{c^3} \frac{1}{m^2} \left(\frac{d\vec{p}}{dt} \right)^2} \qquad F = \frac{dp}{dt} = \frac{dE/c}{d(x/c)} = \frac{dE}{dx}$$

$$\boxed{P = \frac{2}{3} \frac{e^2}{m^2 c^3} \left(\frac{dE}{dx} \right)^2}$$

$$\frac{dE}{dt} = v \frac{dE}{dx} \longrightarrow \frac{P}{(dE/dt)} = \frac{2}{3} \frac{e^2}{m^2 c^3} \frac{1}{v} \frac{dE}{dx} = \frac{2}{3} \frac{(e^2/mc^2)}{mc^2 \left(\frac{v}{c}\right)} \frac{dE}{dx}$$

<u>electron</u>, $\frac{mc^2}{(e^2/mc^2)} = \frac{(0.5\ \text{MeV})}{(2.8\ \text{fm})} = \frac{10^6\ \text{eV}}{10^{-15}\ \text{m}} = 10^{21}\ \frac{\text{eV}}{\text{m}}$.

$$\frac{(\text{TeV})}{(\text{km})} = \frac{10^{12}\ \text{eV}}{(10^3\ \text{m})} = 10^9\ \frac{\text{eV}}{\text{m}}.$$

"parasitic radiation" not a problem for ILC.

$\dot{\vec{\beta}} \perp \vec{\beta}$ synchrotron radiation.

$$\frac{1}{c}\frac{dE}{d\tau} \propto \left|\frac{d\vec{p}}{d\tau}\right| = |\gamma \omega \vec{p}| = \gamma \frac{v}{\rho} \cdot p = \gamma\beta \frac{c}{\rho} \cdot p.$$

$$P = \frac{2}{3}\frac{e^2}{m^2c^3} \cdot \gamma^2\omega^2 p^2 = \frac{2}{3}\frac{e^2}{m^2c^3} \cdot \gamma^2 \left(\frac{\beta^2 c^2}{\rho^2}\right)\left(\gamma^2 m^2 \beta^2 c^2\right)$$

$$P = \frac{2}{3}\frac{e^2 c}{\rho^2} \cdot \gamma^4 \beta^4$$

$$(\delta E)/\text{turn} = \frac{2\pi}{\omega} \cdot P = \left(\frac{4\pi}{3}\right)\frac{e^2}{\rho} \cdot \gamma^4 \beta^3$$

(LHC) (proton). $2\pi\rho = 27$ km $\quad \rho = 4.3$ km

$E = 13$ TeV $\quad mc^2 = 938$ MeV, $\quad \gamma \simeq 13,800 \quad \beta \simeq 1$.

$$\frac{4\pi}{3}\frac{e^2}{\rho}\gamma^4 = \left(\frac{4\pi}{3}\right)\left(3.3\times10^{-13}\,\text{eV}\right)\left(3.7\times10^{16}\right) = 12\text{ keV}.$$

order 10 keV per proton per turn

10^{11} proton / "bunch" $\quad 10^{14} \cdot 10\,\text{keV} \cdot \left(\frac{300{,}000\text{ km/s}}{30\text{ km}}\right)$

2808 "bunches"

$$= 10^{19}\text{ keV/s} = 1\text{ kW}.$$

$$\left(\frac{m_p}{m_e}\right)^4 = 1835^4 = 1.1 \cdot 10^{13}$$

microscopically. angular distribution

$$\frac{dP}{d\Omega} = r^2 \hat{r}\cdot\vec{S} = \frac{e^2}{4\pi c}\left|\frac{\hat{n}\times\left((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right)}{(1-\hat{n}\cdot\vec{\beta})^3}\right|^2_{t'_r = t-\frac{1}{c}R}$$

This is rate at $\vec{x}$ emitted at earlier time.

travel time depends on distance (growing).

$$P'_{source} = \frac{dE}{dt'} = \frac{dE}{dt}\cdot\frac{dt}{dt'} = P\,\frac{d}{dt'}\left(t' + \frac{1}{c}(R^2)^{1/2}\right)$$

$$\rightarrow 1 + \frac{1}{c}\left(\frac{1}{2}\right)(R^2)^{-1/2}\cdot\left(2\vec{R}\cdot\frac{d\vec{R}}{dt'}\right)$$

$$= 1 + \frac{1}{c}\cdot\frac{\vec{R}}{R}\cdot\left(\frac{d\vec{R}}{dt'}\right) = 1 - \hat{n}\cdot\vec{\beta}$$

$$\left.\frac{dP}{d\Omega}\right|_{source} = \frac{e^2}{4\pi c}\cdot\frac{\left|\hat{n}\times\left((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right)\right|^2}{(1-\hat{n}\cdot\vec{\beta})^5}$$

Case. $\vec\beta \parallel \dot{\vec\beta}$ (linear). $\vec\beta \times \dot{\vec\beta} = 0$

$$\frac{dP'}{d\Omega} = \frac{e^2}{4\pi c}\cdot\frac{\left|\hat n\times(\hat n\times\dot{\vec\beta})\right|^2}{(1-\hat n\cdot\vec\beta)^5} = \frac{e^2\dot v^2}{4\pi c^3}\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

gets big when $\beta \simeq 1$ $(\gamma \gg 1)$. $\cos\theta \simeq 1$. $(\theta \ll 1)$

$$\gamma^2 = \frac{1}{1-\beta^2} \rightarrow \beta^2 = 1 - \frac{1}{\gamma^2} \quad \boxed{\beta = 1 - \left(\frac{1}{2\gamma^2}\right)}$$

$$\boxed{\cos\theta \simeq 1 - \tfrac{1}{2}\theta^2} \qquad \sin\theta \simeq \theta$$

$$\frac{dP'}{d\Omega} = \frac{e^2\dot v^2}{4\pi c^3}\cdot\frac{\theta^2}{\left(1-\left(1-\frac{1}{2\gamma^2}\right)\left(1-\frac{1}{2}\theta^2\right)\right)^5}$$

$$\boxed{\frac{dP'}{d\Omega} = \frac{e^2\dot v^2}{4\pi c^3}\frac{\theta^2}{\left(\frac{1}{2\gamma^2}+\frac{1}{2}\theta^2\right)^5}}$$

peaks at $\boxed{\theta = \frac{1}{2\gamma}}$ $\quad$ max $= \frac{8192}{3125}\gamma^8$.

$$\int_{-1}^{1} d\mu.\ \frac{(1-\mu^2)}{(1-\beta\mu)^5} = \int_{1-\beta}^{1+\beta}\frac{dz}{z^5}\left(1-\left(\frac{1-z}{\beta}\right)^2\right) = \frac{4}{3}\frac{1}{(1-\beta^2)^3}$$

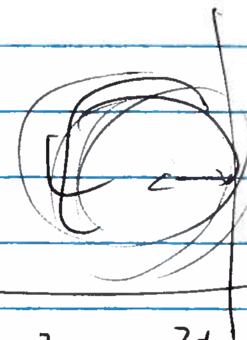
$$\int_{-\theta}^{\theta} d\theta\ \frac{\theta^2}{\left(\frac{1}{2\gamma^2}+\frac{1}{2}\theta^2\right)^5} = \frac{4}{3}\gamma^6$$

$$\boxed{P' = \frac{2e^2}{3}\frac{\dot v^2}{c^3}\gamma^6}\ !$$

$\dot{\vec{\beta}} \perp \vec{\beta}$

let $\vec{\beta} = \beta \hat{t}$

$\dot{\vec{\beta}} = -\dot{\beta} \hat{x}$



$$\boxed{\frac{dP'}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{1}{(1-\beta\cos\theta)^3} \left(1 - \frac{\sin^2\theta \cdot \cos^2\phi}{\gamma^2 (1-\beta\cos\theta)^2} \right)}$$

(14.44)

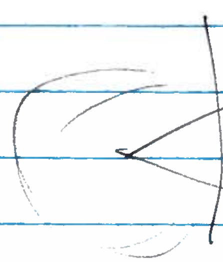
$$\simeq \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{1}{\left(\frac{1}{2\gamma^2} + \frac{1}{2}\theta^2\right)^3} \left(1 - \frac{(\theta/\gamma)^2 \cdot \cos^2\phi}{\left(\frac{1}{2}\gamma^2 + \frac{1}{2}\theta^2\right)^2} \right)$$

peaks again @ $\theta \sim \pm \frac{1}{\gamma}$

$$\int d\Omega \longrightarrow P' = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3} \gamma^4$$

Problem 14.15 $\quad \omega = m\omega_0$

$$\boxed{\frac{dP_m}{d\Omega} = \frac{e^2 \omega_0^4 R^2}{2\pi c^3} m^2 \left[J_m'^2 (m\beta\sin\theta) + \frac{\cot^2\theta}{\beta^2} J_m^2 \right]}$$



$\theta = +\frac{1}{\gamma} \qquad \omega t = \frac{vt}{R} = +\frac{1}{\gamma}$

$\theta = -\frac{1}{\gamma} \qquad \omega t = -\frac{1}{\gamma}$

$$\Delta L = c(2\Delta t) - v(2\Delta t)$$

$$= (c-v) \cdot \frac{2R}{\gamma v}$$

$$= \frac{2R}{\gamma}\left(\frac{1}{\beta} - 1\right) = \frac{R}{\gamma^3} //$$