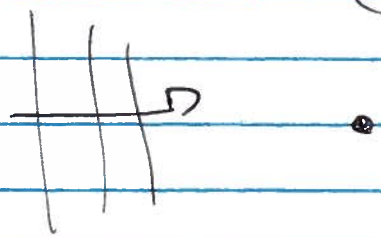
4/23/2018 Scattering from charged particle

(Thomson scattering) §14.8



$$\vec{E}_0 = E_0 \vec{\varepsilon}_0 \, e^{i(\vec{k}_0\cdot\vec{x} - \omega t)}$$

$$m\ddot{\vec{x}} = m\dot{\vec{v}} = e\vec{E}_0$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\left|\hat{r}\times(\hat{r}\times\ddot{\vec{x}})\right|^2 = \frac{e^2}{4\pi c^3}\left|\hat{r}\times\left(\hat{r}\times\left(\frac{e\vec{E}_0}{m}\right)\right)\right|^2$$

$$\vec{S}_{in} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\vec{E}_0\times(\hat{k}_0\times\vec{E}_0) = \frac{c}{4\pi}\hat{k}_0|\vec{E}_0|^2$$

$$\langle\cdot\rangle \to \frac{1}{2}\mathrm{Re}(\vec{E}^*\cdot\vec{E}) \text{ (both).}$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{e^2}{4\pi c^3}\cdot\frac{e^2}{m^2}\left|\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\right|^2 \cancel{|\vec{E}_0|^2}}{\frac{c}{4\pi}\cancel{|\vec{E}_0|^2}}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2\left|\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\right|^2}$$

cf. (14.122)

$$|\hat{r}\times\vec{\varepsilon}_0| = (\hat{r}\times\vec{\varepsilon}_0^{*})\cdot(\hat{r}\times\vec{\varepsilon}_0) = \hat{r}\cdot\left(\vec{\varepsilon}_0^{*}\times(\hat{r}\times\vec{\varepsilon}_0)\right)$$

$$= \hat{r}\cdot\left(\hat{r}\,(\underbrace{\vec{\varepsilon}_0^{*}\cdot\vec{\varepsilon}_0}_{1}) - \vec{\varepsilon}_0(\hat{r}\cdot\vec{\varepsilon}_0^{*})\right) = 1-(\hat{r}\cdot\vec{\varepsilon}_0)^2$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta .$$

$$\vec{\varepsilon}_0 = \hat{x}\cos\phi_0 + \hat{y}\sin\phi_0 .$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2\left(1-\sin^2\theta\cdot\cos^2(\phi-\phi_0)\right)}$$

strongest @ poles, $\sin\theta\to 0$.

unpolarized. $\langle\cos^2(\phi-\phi_0)\rangle = \frac{1}{2}$

(14.125) $$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2\frac{1}{2}(1+\cos^2\theta)}$$

(14.126) $$\boxed{\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2}$$

polarized detection |

$$\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\cdot\hat{\theta} = -\cos\theta\,\cos(\phi-\phi_0)$$

$$\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\cdot\hat{\phi} = \sin(\phi-\phi_0)$$

J.J. Thomson . 1906 Nobel prize.

$$\frac{e^2}{mc^2} = r_0 = \frac{e^2/4\pi\varepsilon_0}{mc^2} = 2.8 \times 10^{-15}\ m$$

fm.

$$\sigma_T = 6.65 \times 10^{-25}\ cm^2 = 0.665 \text{ "barn"}$$

QFT. $\hbar\omega \ll mc^2$. $\left(1 - \frac{2\hbar\omega}{mc^2}\right)$ Compton scattering

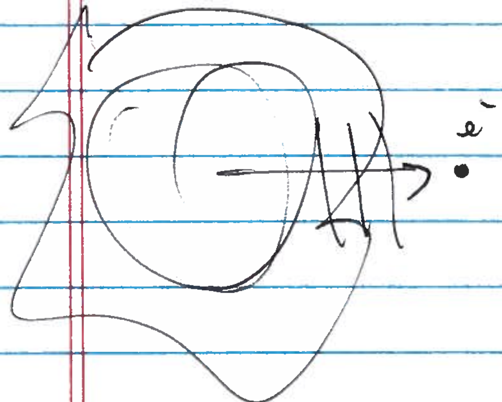
$\hbar\omega \gg mc^2$. $\frac{3}{4}\frac{mc^2}{\hbar\omega} \log\left(\frac{2\hbar\omega}{mc^2}\right)$ Klein-Nishina

CMB: $H^+ + e^- \leftrightarrow \gamma\gamma$.

opaque until. $T \sim 3000K$.

$t \sim 380{,}000\ y$.

$z = 1089$



$$P_{scatt} = \left(\frac{dE}{dt}\right)_{scatt} = \sigma_T \cdot \left(\frac{dP}{da}\right)_{wave} = \sigma_T \cdot \frac{L_\odot}{4\pi R^2}$$

$E = \hbar\omega$ $\quad$ $\vec{p} = \hbar\vec{k}$

$$\frac{\vec{g}}{u} = \frac{(\vec{E}\times\vec{B})/4\pi c}{(E^2+B^2)/8\pi} = \frac{\vec{k}}{\omega}$$

$$\left(\frac{d\vec{p}}{dt}\right)_{scatt} = \frac{\vec{k}}{\omega} \cdot \left(\frac{dE}{dt}\right)_{scatt} = \frac{\hat{r}}{c} \cdot \sigma_T \cdot \frac{L}{4\pi R^2}$$

$$\boxed{\vec{F}_{rad} = \hat{r} \cdot \frac{L}{4\pi R^2} \cdot \frac{\sigma_T}{c}}$$

Blast off electrons → $\vec{F} = -\frac{GM m_e}{R^2} + \frac{Qe}{R^2} + \frac{L\sigma_T}{4\pi R^2}$

proton $\vec{F} = +\frac{Qe}{R^2} - \frac{GM m_p}{R^2}$

$m_e \ll m_p$ protons kicked off if $\boxed{\frac{L}{4\pi R^2}\frac{\sigma_T}{c} > \frac{GM m_p}{R^2}}$

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max. $\boxed{L < L_E = \dfrac{4\pi G M m_p c}{\sigma_T}}$ Eddington.

Sun : $M_\odot = 2\times10^{33}\,g$

$L_\odot = 4\times10^{33}\,erg\,s^{-1}$

$L_E = 1.26\times10^{38}\,erg\,s^{-1}$ $\boxed{L/L_E = 3\times10^{-5}}$ main sequence.

O*. $M > 16\,M_\odot$

Wolf-Rayet. $M > 20\,M_\odot$ $T \simeq 200{,}000\,K$!

Lifetime of a star.

$\Delta E = \varepsilon M c^2$ (H→He) $\varepsilon = \dfrac{28\,MeV}{4\,GeV} = 0.007$

$L = Y L_E$.

$$\tau = \frac{\varepsilon \cancel{M} c^2}{Y}\frac{\sigma_T}{4\pi G \cancel{M} m_p c} = \frac{\varepsilon}{Y}\left(\frac{8\pi}{3}\right)\frac{(e^2/m_e c^2)^2 \cdot c}{4\pi G m_p}$$

$$= \frac{2}{3}\frac{\varepsilon}{Y}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{\hbar c}{G m_p m_e}\right)\left(\frac{\hbar}{m_e c^2}\right)$$

$$= \frac{2}{3}\frac{\varepsilon}{Y}\left(\frac{1}{137.036}\right)^2\left(3.11\times10^{41}\right)\left(1.3\times10^{-21}\,s\right)$$

$$= \frac{\varepsilon}{Y}\left(4.5\times10^{8}\,Y\right) = \frac{3\,MY}{(Y/E)}$$

Sun → 100 Gy !

Back to Chapter 12

Classical. $S = \int dt \, L(q_j, \dot{q}_j)$ $\qquad L = \sum_j \frac{1}{2}\dot{q}_j^2 - V.$

Field: $\quad q_j(t) \longrightarrow \phi(\vec{x}, t) \qquad \sum_j \longrightarrow \int d^3x.$

$$S = \int dt \int d^3x \, \mathcal{L}[\phi(\vec{x},t), \partial_\mu \phi].$$

Lorentz invariant. $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}|\vec{\nabla}\phi|^2 - V(\phi)$

$\hookrightarrow \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi.$

$$\delta S = S[\phi + \delta\phi] - S[\phi]$$

$$= \int d^4x \cdot \left[\frac{\partial \mathcal{L}}{\partial \phi} \cdot \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \cdot \delta(\partial_\mu \phi) \right.$$

$$\delta(\partial_\mu \phi) = \partial_\mu [\phi + \delta\phi] - \partial_\mu \phi = \partial_\mu (\delta\phi).$$

$$\delta S = \int d^4x \cdot \left[\frac{\partial \mathcal{L}}{\partial \phi} \cdot \delta\phi + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \cdot \partial_\mu (\delta\phi)}_{\text{parts.}} \right]$$

$$= \int d^4x \cdot \left(\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \right) \cdot \delta\phi = 0$$

$\delta\phi$ arbitrary $\boxed{\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0}$ (12.83)

$$\mathcal{L} = \frac{1}{2}\eta^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - V(\phi).$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{1}{2}(\partial_\beta\phi)\eta^{\mu\beta} + \frac{1}{2}(\partial_\alpha\phi)\eta^{\alpha\mu} = \underline{\partial^\mu\phi}$$

$$\underline{\frac{\partial\mathcal{L}}{\partial\phi} = -\frac{\partial V}{\partial\phi}}$$

$$\partial_\mu(\partial^\mu\phi) = \boxed{\Box^2\phi = -\frac{\partial V}{\partial\phi}}$$

$$V = \frac{1}{2}m^2\phi^2 \qquad \boxed{\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi = -m^2\phi}$$

Klein-Gordon equation

$$\phi \sim e^{i(\vec{k}\cdot\vec{x} - \omega t)}. \qquad -\omega^2 + |\vec{k}|^2 = -m^2$$

$$\boxed{(\hbar\omega)^2 = (\hbar c\vec{k})^2 + (mc^2)^2}$$