1/6/2015

$\boxed{\text{Chapter 7}}$ E.M. waves.

$$\vec{\nabla}\cdot\vec{E} = \cancel{\rho/\varepsilon} = 0 \qquad \text{no sources.}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \cancel{\mu\vec{J}} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) - \nabla^2\vec{E}$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}} \qquad \boxed{\nabla^2\vec{B} = \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}}$$

$$\mu = k_m\mu_0 \qquad \varepsilon = k_e\varepsilon_0 \qquad \mu\varepsilon = \frac{k_e k_m}{c^2} = \frac{n^2}{c^2}$$

Separation of variables

$$\boxed{\begin{aligned}\vec{E} &= \vec{E}_0\, e^{i(\vec{k}\cdot\hat{x} - \omega t)}\\ \vec{B} &= \vec{B}_0\, e^{i(\vec{k}\cdot\hat{x} - \omega t)}\end{aligned}}$$

$$|\vec{k}|^2 = \mu\varepsilon\omega^2 = \frac{n^2\omega^2}{c^2}$$

(7.8). $\vec{\mathcal{E}}$ $\vec{B}$

constant n. $f'' = \frac{n^2}{c^2}\ddot{f}$ $\qquad \boxed{v^2 = \frac{c^2}{n^2}}$

$f(x - vt)$

Co ts of wave equations

$\vec{\nabla}\cdot\vec{E} = i\vec{k}\cdot\vec{E} = 0$ $\quad \vec{k}\cdot\vec{E}_0 = 0 \quad \vec{k}\cdot\vec{B}_0 = 0$

Fields $\vec{E}_0, \vec{B}_0$ $\perp$ wave number $\vec{k}$ "transverse"

$\vec{\nabla}\times\vec{E} = i\vec{k}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = +i\omega\vec{B}$ $\quad \vec{k}\times\vec{E}_0 = \omega\vec{B}_0$

write $\vec{k} = k\hat{k}$ $\quad \hat{k}\times\vec{E}_0 = \frac{\omega}{k}\vec{B}_0$ $\quad \hat{k}\times\vec{E}_0 = \frac{c}{n}\vec{B}_0$

$(\hat{E}, \hat{B}, \hat{k}) \to (\hat{x}, \hat{y}, \hat{z})$

Jackson likes $\vec{H}$. ~~in vacuum~~ space. $\vec{B} = \mu\vec{H}$

$\vec{\nabla}\times\vec{E} = i\vec{k}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} = +i\mu\omega\vec{H}$

$\hat{k}\times\vec{E}_0 = \frac{\mu\omega}{k}\vec{H}_0 = \frac{\mu}{\sqrt{\mu\varepsilon}}\vec{H}_0 = \sqrt{\frac{\mu}{\varepsilon}}\vec{H}_0 = Z\vec{H}_0$

$\sqrt{\frac{\mu}{\varepsilon}} = Z$ $\qquad \sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 = 4\pi\sqrt{\frac{\mu_0}{4\pi}}\sqrt{\frac{1}{4\pi\varepsilon_0}}$

impedance.

$= 4\pi\cdot\sqrt{10^{-7}\cdot 9\cdot 10^{9}}$

$[E] = V/m$ $\qquad = 4\pi\cdot(2.99792458)\cdot 10\ \Omega$

$[H] = A/m.$ $\qquad Z_0 = 376.730313\ \Omega$

$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}\left[\vec{E} \times \vec{H}^*\right]$$ ← Learn this

$$= \frac{1}{2} \mathrm{Re}\left[\vec{E}_0 \cancel{e^{i(\vec{k}\cdot\vec{x}-\omega t)}} \times \left(\sqrt{\frac{\varepsilon}{\mu}}\, \hat{k} \times \vec{E}_0^* \cancel{e^{-i(\vec{k}\cdot\vec{x}-\omega t)}}\right)\right]$$

$$= \frac{1}{2} \mathrm{Re}\left[\sqrt{\frac{\varepsilon}{\mu}}\; \hat{k}\,(\vec{E}_0 \cdot \vec{E}_0^*) - \vec{E}_0^*\,\cancel{(\hat{k}\cdot\vec{E}_0)}\right]$$

$$\boxed{\langle \vec{S} \rangle = \frac{1}{2} \frac{|\vec{E}_0|^2\, \hat{k}}{\mathrm{Re}\sqrt{\mu/\varepsilon}}}$$

wave vector direction = energy flow direction

$$[S] = \frac{[V/m]^2}{\Omega} = \frac{(V^2/\Omega)}{m^2} = \frac{\text{Power}}{\text{Area}}$$

$$\langle u \rangle = \mathrm{Re}\left[\frac{1}{4}\varepsilon|\vec{E}|^2 + \frac{1}{4}\mu|\vec{H}|^2\right]$$

$$\frac{1}{4}\mu \cdot \left(\frac{\varepsilon}{\mu}\right)|\vec{E}|^2$$

equipartition

$$\boxed{\langle u \rangle = \frac{1}{2}\,\mathrm{Re}\,\varepsilon)\,|\vec{E}_0|^2}$$

$$\frac{\text{rate}}{\text{density}} = \frac{\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\,|E_0|^2}{\frac{1}{2}\,\varepsilon\,|E_0|^2} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n}$$

$\underline{\omega}$ is $\underline{\text{real}}$ complex $\underline{n}$ $\Rightarrow$ complex $\underline{\vec{k}}$

$$\vec{k} = \vec{k}_r + i\vec{k}_i \quad \leftarrow \text{decay } e^{-\vec{k}_i \cdot \vec{x}}$$

$$|\vec{k}|^2 = k_r^2 - k_i^2 + 2i(\vec{k}_r \cdot \vec{k}_i) = \frac{n^2\omega^2}{c^2}$$

Order:

1-wave things before 2-wave things

↑ dispersion, polarization ↑ reflection

to begin: 1-d, $\hat{k} = \hat{x}$.

$$\psi(x, t_0) = \psi_0(x) = \int \frac{dk}{2\pi} A(k)\, e^{ikx}$$

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{i(kx - \omega t)}$$

$$A(k) = \int dx\, \psi_0(x)\, e^{-ikx}$$

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If: $\frac{\omega}{k} = v$ = <u>constant</u>. $\omega = vk$.

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{ik(x-vt)} = \psi_0(x-vt)$$

"moving wave",

@ given ω. $e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)} = e^{ik(x-v_{ph}t)}$

$$\boxed{v_{ph} = \frac{\omega}{k}} = \text{phase velocity.}$$

<u>Suppose</u>. $\omega(k)$ <u>not</u> constant
$A(k)$ peaked near k_0

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{i\left(kx - \left(\omega_0 + \frac{d\omega}{dk}\big|_0 (k-k_0) + \dots\right)t\right)}$$

$$= e^{-i\omega_0 t}\, e^{ik_0\left(\frac{d\omega}{dk}\right)_0 t} \int \frac{dk}{2\pi} A(k)\, e^{i\left(k - \left(\frac{d\omega}{dk}\right)_0 t\right)}$$

$$\boxed{\psi(x,t) = e^{-i(\omega_0 - k_0 v_g)t}\, \psi_0(x - v_g t)}$$

$$\boxed{v_{group} = \frac{d\omega}{dk}\Big|_{k_0}}$$ "group velocity"

$\omega = k^p$. $\frac{k}{\omega}\frac{d\omega}{dk} = p$. $e^{-i(1-p)\omega_0 t}$