1/8/2016 $\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)$

$$\bar{k} = \frac{1}{2}(k_1 + k_2)$$

$$\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$$

$$\Delta k = k_1 - k_2$$

$$\Delta\omega = \omega_1 - \omega_2$$

$$\cos\left(\bar{k}x - \bar{\omega}t + \frac{1}{2}(\Delta k x - \Delta\omega t)\right) + \cos\left(\bar{k}x - \bar{\omega}t - \frac{1}{2}(\Delta k x - \Delta\omega t)\right)$$

$$= 2 \cdot \underbrace{\cos(\bar{k}x - \bar{\omega}t)}_{\text{(carrier)}} \cdot \underbrace{\cos\left(\frac{1}{2}(\Delta k x - \Delta\omega t)\right)}_{\text{(modulation)}}$$

Fun

(2)

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{i(kx-\omega t)}$$

constant phase: $kx - \omega t = \phi_0$ (single wave).

$x - \left(\frac{\omega}{k}\right)t$ = constant

$x = x_0 + \left(\frac{\omega}{k}\right)t$.

$\boxed{\omega/k = v_{ph}}$

$\boxed{\frac{d\omega}{dk} = v_{group}}$ (last time).

$k^2 = \mu\epsilon\omega^2$ $\quad k^2 = n^2\frac{\omega^2}{c^2}$ $\quad \boxed{k = \frac{n\omega}{c}}$

$\boxed{v_{ph} = \frac{\omega}{k} = \frac{c}{n}}$

$$\frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{c}{n + \omega\left(\frac{dn}{d\omega}\right)}$$

"typically" $n > 1$ (lenses).

$\frac{dn}{d\omega} > 0$ "normal"

$\boxed{v_g < v_{ph} < c}$

$\frac{dn}{d\omega} < 0$ "anomalous dispersion"

Next order. ← group velocity ← dispersion

$$\omega \simeq \omega(k_0) + \left(\frac{d\omega}{dk}\right)_0 \cdot \Delta k + \frac{1}{2}\left(\frac{d^2\omega}{dk^2}\right)_0 (\Delta k)^2 + \cdots$$

← JDJ "α" (7.95)

$$\ln A(k) \simeq \ln A(k_0) + \cancel{\left(\frac{d \ln A}{dk}\right)_0} \cdot \Delta k + \frac{1}{2}\left(\frac{d^2 \ln A}{dk^2}\right) \cdot \Delta k^2$$

peaked. ← $-L^2$

take $\boxed{A(k) = A \cdot e^{-\frac{1}{2}L^2(k-k_0)^2}}$

(can do the integrals.)

$t=0$.

$$E_0(t) = \int \frac{dk}{2\pi} A(k) e^{ikx} = \int \frac{dk}{2\pi} A\, e^{-\frac{1}{2}L^2(k-k_0)^2} e^{ik_0x} e^{i(k-k_0)x}$$

$$= A e^{ik_0x} \int \frac{d(\Delta k)}{2\pi} \exp\left[-\frac{1}{2}L^2\left(\underbrace{(\Delta k)^2 - \frac{2ix}{L^2}\cdot\Delta k + \left(\frac{ix}{L^2}\right)^2}_{\left(\Delta k - \frac{ix}{L^2}\right)^2} - \left(\frac{ix}{L^2}\right)^2\right)\right]$$

$$= A e^{ik_0x} e^{-\frac{1}{2}L^2(-)\left(\frac{ix}{L^2}\right)^2} \underbrace{\int \frac{d(\Delta)}{2\pi} \exp\left(-\frac{1}{2}L^2(\Delta)^2\right)}_{\frac{1}{2\pi}\cdot\sqrt{2\pi}\cdot\frac{1}{L} = \frac{1}{\sqrt{2\pi}L}}$$

$$\boxed{E_0(x) = \frac{A}{\sqrt{2\pi}\cdot L} e^{ik_0x} e^{-\frac{1}{2}x^2/L^2}}$$ (7.92) $\cos k_0x$.

JDJ. $\cos k_0 x$ peaks at $\pm k_0$ left. right symmetric

$$E(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{i(kx - \omega t)}$$

$A(k) \hookrightarrow e^{-\frac{1}{2}L^2 \Delta k^2}$; $\omega \to \omega_0 + \omega_0' \Delta k + \frac{1}{2}\omega_0'' \Delta k^2 + \dots$

$$= \int \frac{dk}{2\pi} A\, e^{-\frac{1}{2}L^2 \Delta k^2} \exp\Big[i(k_0 + \Delta k)x - i\omega_0 t - i\omega_0' \Delta k\, t - \tfrac{1}{2} i \omega_0'' \Delta k^2 t \Big]$$

$$= \frac{A}{2\pi} e^{i(k_0 x - \omega_0 t)} \int d(\Delta k) \exp\big(i \Delta k (x - \omega_0' t)\big) \times \exp\Big[-\tfrac{1}{2}(L^2 + i\omega_0'' t)\Delta k^2\Big]$$

center was $x = 0$ now $x = v_g t$ $\underline{(x - \omega_0' t)}$

width was L^2 now. $\underline{L^2 + i\omega_0'' t}$,

$$E = \frac{A\, e^{i(k_0 x - \omega_0 t)}}{\sqrt{2\pi}\sqrt{L^2 + i\omega_0'' t}} \exp\left(-\frac{1}{2} \frac{(x - v_g t)^2}{(L^2 + i\omega_0'' t)} \right)$$

$$\frac{1}{L^2 + i\omega_0'' t} = \frac{L^2 - i\omega_0'' t}{(L^2)^2 + (\omega_0'' t)^2}$$

Imaginary $\to$ phase.
Real $\to$ gaussian envelope $\to$ width.

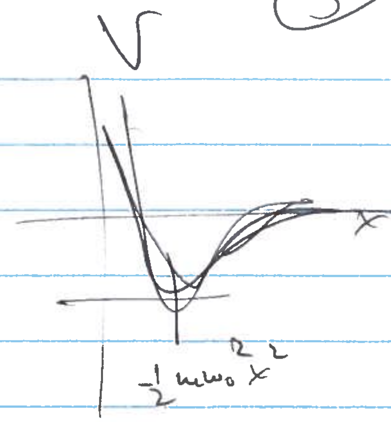
$$(\text{width})^2 = \frac{(L^2)^2 + (\omega_0'' t)^2}{L^2} = L^2 + \left(\frac{\omega_0'' t}{L^2}\right)^2$$

(7.99) (7.100)

narrower $L^2 \to$ broader $(\Delta k)^2 \to$ widens ~~[illegible]~~ earlier.

Simple ("toy") model

harmonic oscillator atom



$$\vec F = -k\vec x - b\vec v + e\vec E$$

$$m(\ddot{\vec x} + \gamma\dot{\vec x} + \omega_0^2 \vec x) = e\vec E = e\vec E_0 e^{-i\omega t},$$

driven $\longrightarrow$ $\vec x = \vec x_0 e^{-i\omega t}$ (~~transient~~)

$$m(-\omega^2 - i\gamma\omega + \omega_0^2)\,\vec x_0 = e\vec E_0$$

$$\vec x_0 = \frac{e\vec E_0/m}{\omega_0^2-\omega^2-i\gamma\omega} \longleftarrow \frac{(\omega_0^2-\omega^2)+i\gamma\omega}{(\omega_0^2-\omega^2)^2+(\gamma\omega)^2}$$

$\vec p = e\vec x$. $\vec P = N_{atm}\, e\, \vec p = \frac{Ne^2\vec E_0/m}{\omega^2\ldots} = \varepsilon_0\chi\vec E$

$$\boxed{\frac{\varepsilon}{\varepsilon_0} = 1+\chi = 1 + \frac{Ne^2}{m\varepsilon_0}\,\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}}$$ 7.56

multiple modes

$$\boxed{n^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_k \frac{f_k}{\omega_k^2-\omega^2-i\gamma_k\omega}}$$

$\sum f_k = Z$. $N \cdot Z = n_e$