1/11/2016

$$\frac{\varepsilon}{\varepsilon_0} \to 1 + \frac{Ne^2}{m\varepsilon_0} \sum_k \frac{f_k}{\omega_k^2 - \omega^2 - i\gamma_k \omega}$$

$\sum f_k = Z$

$NZ = n_e$

$\omega \to 0$ $$\frac{\varepsilon}{\varepsilon_0} \to 1 + \sum_k \frac{f_k}{\omega_k^2} \frac{Ne^2}{m\varepsilon_0}$$

Static dielectric constant

Free electrons $\omega_0 \to 0$ lowest.

$$\frac{\varepsilon}{\varepsilon_0} \to 1 + \sum_{k \neq 0} \ldots + \frac{Ne^2}{m\varepsilon_0} \frac{1}{(-\omega^2 - i\gamma\omega)}$$

$$= K + \frac{Ne^2}{m\varepsilon_0} \frac{1}{\omega} \frac{i}{\gamma - i\omega}.$$ diverges as $\omega \to 0$

$$\vec{\nabla} \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t} = (\mu_0)\left(K\varepsilon_0 + \frac{ne^2}{m} \frac{i}{\omega(\gamma - i\omega)}\right)(-i\omega\vec{E})$$

$$= \frac{n\mu_0 e^2}{m\gamma} \vec{E} \underbrace{- i\omega(\mu\varepsilon)\vec{E}}_{\mu\varepsilon \frac{\partial \vec{E}}{\partial t}.}$$

$\mu(\sigma\vec{E})$

$\vec{J} = \sigma\vec{E}$ Conductivity

$\mathrm{Re}\,\varepsilon \to K\varepsilon_0$

$\mathrm{Im}\,\varepsilon \to \frac{i\sigma}{\omega}$

at force level $\vec{F} = e\vec{E} - m\gamma\vec{v}$

steady state. $\gamma\vec{v} = e\vec{E}$

$$\boxed{\vec{J} = ne\vec{v} = \frac{ne^2\vec{E}}{\gamma m}}$$

$\boxed{\omega \to \text{big}}$

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_k \frac{f_k}{\omega_k^2 - \omega^2 - i\gamma\omega}$$

$$= 1 - \frac{n_e e^2}{m\varepsilon_0}\frac{1}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

$\boxed{\omega_p^2 = \frac{n_e e^2}{m\varepsilon_0}}$ "plasma frequency"

$$k^2 = \mu\varepsilon\omega^2 = \mu_0\varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)\omega^2 = \frac{1}{c^2}(\omega^2 - \omega_p^2)$$

$$\boxed{\omega^2 = \omega_p^2 + c^2k^2}$$

$\cancel{2\omega = c^2 \cdot 2k\frac{d\omega}{dk}}$

$$2\omega\frac{d\omega}{dk} = 2c^2 \cdot k \qquad \boxed{\frac{\omega}{k}\cdot\frac{d\omega}{dk} = c^2}$$

$\frac{\omega}{ck} > 1$ $\quad \frac{1}{c}\frac{d\omega}{dk} < 1$ $\quad \frac{\omega}{k} > c$ $\quad \frac{d\omega}{dk} < c$

polarization (the other kind).

$\vec\nabla\cdot\vec E=0$ $\quad \vec k\cdot\vec E=0 \quad$ 2 d.f. $\quad \vec E = (E_1\vec\varepsilon_1 + E_2\vec\varepsilon_2)\, e^{i(\vec k\cdot\vec x-\omega t)}$

$\hat k=\hat z \quad \vec\varepsilon_1=\hat x \quad \vec\varepsilon_2=\hat y \quad \vec\varepsilon_i\cdot\hat k=0$

$\vec\varepsilon_i\cdot\vec\varepsilon_j^{(*)}=\delta_{ij}$

That simple, wouldn't dwell on it.

Take $E_1=E_0 \quad E_2=iE_0 \quad \vec E = E_0(\vec\varepsilon_1+i\vec\varepsilon_2)\, e^{i(\vec k\cdot\vec x-\omega t)}$

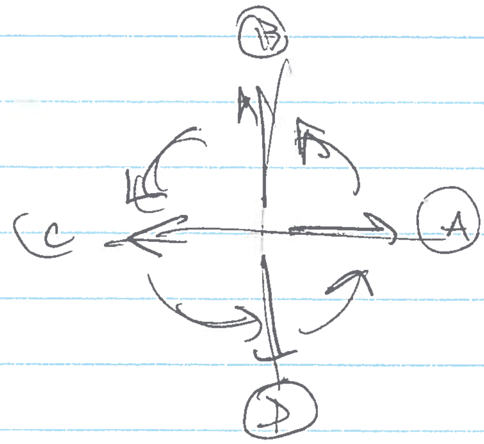
Fixed $\vec x$ ($\vec x=0$). $\quad E_0(\vec\varepsilon_1+i\vec\varepsilon_2)\,e^{-i\omega t}$

A phase $=0$, $\quad$ Re $\to \vec\varepsilon_1$

B phase $=\frac{\pi}{2}$ $\quad e^{-i\frac{\pi}{2}}=-i \quad$ Re $\to \vec\varepsilon_2$

C phase $=\pi$ $\quad e^{-i\pi}=-1 \quad$ Re $\to -\vec\varepsilon_1$

D phase $:\frac{3\pi}{2}$ $\quad e^{-i\frac{3\pi}{2}}=+i \quad$ Re $\to -\vec\varepsilon_2$



$\vec E$ sweeps around circle: circular polarization

normalized. $\quad \vec\varepsilon_\pm = \frac{1}{\sqrt2}(\vec\varepsilon_1\pm i\vec\varepsilon_2)$

$\vec\varepsilon_\pm\cdot\vec\varepsilon_\pm^{*} = 1 = \vec\varepsilon_\pm\cdot\vec\varepsilon_\mp$

$\vec\varepsilon_\pm\cdot\vec\varepsilon_\mp^{*} = 0 = \vec\varepsilon_\pm\cdot\vec\varepsilon_\mp^{*}$

$\vec\varepsilon_i\cdot\vec\varepsilon_j^{*}=\delta_{ij}$

Any $\vec{E}$ can be written in either basis.

$$\vec{E} = (E_1\hat{\varepsilon}_1 + E_2\hat{\varepsilon}_2)\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} = (E_+\vec{\varepsilon}_+ + E_-\vec{\varepsilon}_-)\, e^{i(\vec{k}\cdots}$$

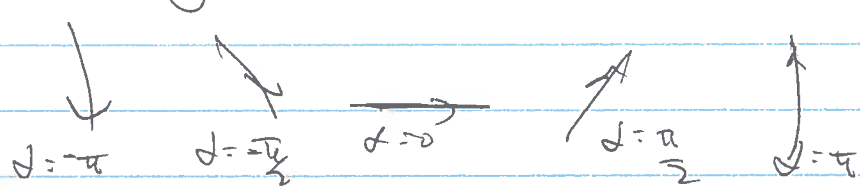
$$\sqrt{E_\alpha = \vec{E}\cdot\vec{\varepsilon}_\alpha^{\,*}} \quad \text{always.}$$

Arbitrary ("elliptical") polarization

$$\vec{E} = E_0\left(\cos\frac{\alpha}{2}\, e^{-i\beta/2}\,\vec{\varepsilon}_1 + \sin\frac{\alpha}{2}\, e^{+i\beta/2}\,\vec{\varepsilon}_2\right)$$

$\beta = 0$ relatively real. → linear polarization

$-\pi < \alpha < \pi$



$\beta = \pm\frac{\pi}{2}$ circular

$(\alpha \to \frac{\pi}{2})\ (\pm)$

in between "elliptical"

Characterize polarization content:

"Stokes parameters"

Stokes Theorem.

$\oint \vec{A} \cdot d\vec{l} = \int d^2 \vec{a} \cdot \nabla \times \vec{A}$

Navier-Stokes equation.

$$s_0 = |\vec{E} \cdot \vec{\varepsilon}_1^*|^2 + |\vec{E} \cdot \vec{\varepsilon}_2^*|^2$$

$$= |E_0|^2 \cdot \cos^2\frac{\alpha}{2} + |E_0|^2 \cdot \sin^2\frac{\alpha}{2} = |E_0|^2$$

$$s_1 = |\vec{E} \cdot \vec{\varepsilon}_1^*|^2 - |\vec{E} \cdot \vec{\varepsilon}_2|^2$$

$$s_2 = 2\,\mathrm{Re}\left[(\vec{E} \cdot \vec{\varepsilon}_1^*)^* (\vec{E} \cdot \vec{\varepsilon}_2^*)\right]$$

(7.27)

$$s_3 = 2\,\mathrm{Im}\left[(\vec{E} \cdot \vec{\varepsilon}_1^*)^* (\vec{E} \cdot \vec{\varepsilon}_2^*)\right]$$

$E_\pm$ (7.28)

$E_\alpha \beta$

$$Q = |E_1|^2 - |E_2|^2$$

$$U = |E_\alpha|^2 - |E_\beta|^2$$

$$V = |E_+|^2 - |E_-|^2$$

$$|\vec{S}|^2 = s_0^2$$