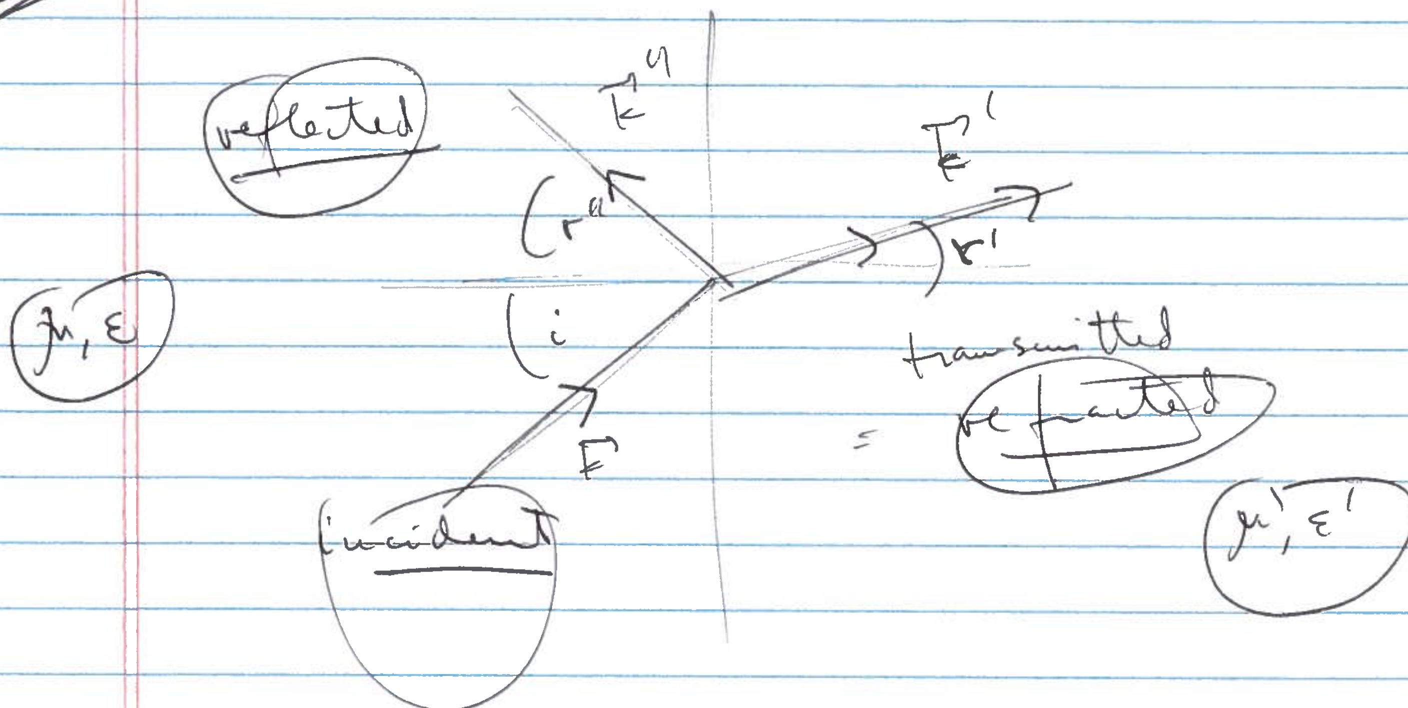
1/13/2016 Two-wave things: reflection from interface



incident
$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$
$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\hat{x} - \omega t)} = \sqrt{\mu\varepsilon}\,\hat{k}\times\vec{E} = \frac{n}{c}\hat{k}\times\vec{E}$$

reflected
$$\vec{E}'' = \vec{E}_0'' e^{i(\vec{k}''\cdot\hat{x} - \omega'' t)}$$
$$\vec{B}'' = \vec{B}_0'' e^{i(\vec{k}''\cdot\hat{x} - \omega t)} = \frac{n}{c}\hat{k}''\times\vec{E}''$$

refracted
$$\vec{B}' = \vec{B}_0' e^{i(\vec{k}'\cdot\hat{x} - \omega t)} = \frac{n'}{c}\hat{k}'\times\vec{E}'$$

$\omega' = \omega'' = \omega$

red ⟶ green

red NO (nonlinear optics)

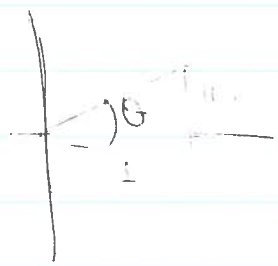
at interface ($z=0$) $\vec{x} = \vec{x}_\parallel$ 2

$\vec{k} = \vec{k}_\parallel + k_\perp \hat{z}$ $\vec{k}\cdot\vec{x} = \vec{k}_\parallel \cdot \vec{x}_\parallel$ for all

Any matching condition $\longrightarrow$

$e^{i\vec{k}_\parallel\cdot\vec{x}_\parallel} \propto e^{i\vec{k}'_\parallel\cdot\vec{x}_\parallel} \propto e^{i\vec{k}''_\parallel\cdot\vec{x}_\parallel}$

same $\vec{x}_\parallel$ $\longrightarrow$ $\vec{k}_\parallel = \vec{k}'_\parallel = \vec{k}''_\parallel$



$k_\perp = k\cos\theta$

$k_\parallel = k\sin\theta$

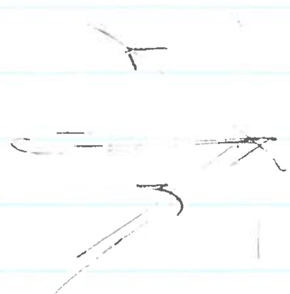
$\Rightarrow$ $\boxed{k\sin i = k'\sin r' = k''\sin r''}$

$k^2 = \mu\epsilon\omega^2 = k''^2$ $\sin i = \sin r''$

$k_z^2 = k^2 - k_\parallel^2 = k^2(1-\sin^2 i)$

$k_z''^2 = k''^2 - k''^2_\parallel = k^2(1-\sin^2 r'')$ $\Big\}$ $k_z'' = \pm k_z$

$k_z'' = -k_z$



$r'' = i$ "specular reflection"

$k_\parallel$ preserved $k_\perp$ reversed

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refracted: $k_\parallel = k\sin i = k'_\parallel = k'\sin r'$

$$\frac{n\omega}{c}\sin i = \frac{n'\omega}{c}\sin r'$$

$$\boxed{n\sin i = n'\sin r'} \rightarrow \boxed{n_1\sin\theta_1 = n_2\sin\theta_2}$$

Snell's law (for any b.c.).

Maxwell: $\Delta\vec{E}_\parallel = 0 \quad \Delta D_\perp = 0 \quad \Delta H_\parallel = 0 \quad \Delta B_\perp = 0$

$$\hat{n}\times\left(\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right) = 0$$

$$\hat{n}\cdot\left(\varepsilon\left(\vec{E}_0 + \vec{E}_0''\right) - \varepsilon'\vec{E}_0'\right) = 0$$

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{k}\times\vec{E} = \omega\vec{B}$

$$\hat{n}\cdot\left(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}_0'' - \vec{k}'\times\vec{E}_0'\right) = 0$$

$$\hat{n}\times\left(\frac{1}{\mu}\vec{k}\times\vec{E}_0 + \frac{1}{\mu}\vec{k}''\times\vec{E}_0'' - \frac{1}{\mu'}\vec{k}'\times\vec{E}_0'\right) = 0$$

6 unknowns $\vec{E}_0', \vec{E}_0''$

8 equations $\longrightarrow$ need some luck

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Geometry let $\hat{n} = \hat{z}$ $\vec{k}_{4A} = \vec{k}'_{4B} = \vec{k}''_{4}$ in x-z plane

$\vec{k} = k\cos i\,\hat{z} + k\sin i\,\hat{x}$

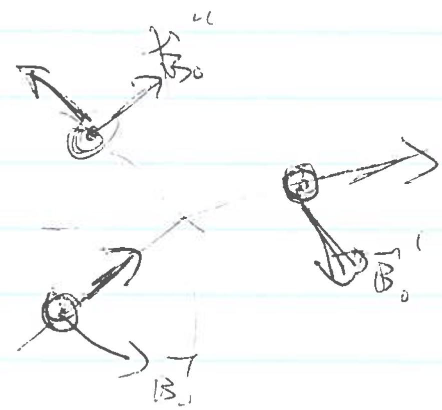
$\vec{k}' = k'\cos r'\,\hat{z} + k'\sin i\,\hat{x}$

$\vec{k}'' = k''\cos r''\,\hat{z} + k''\sin r''\,\hat{x}$

$-k\cos i\,\hat{z}$ all $k\sin i$

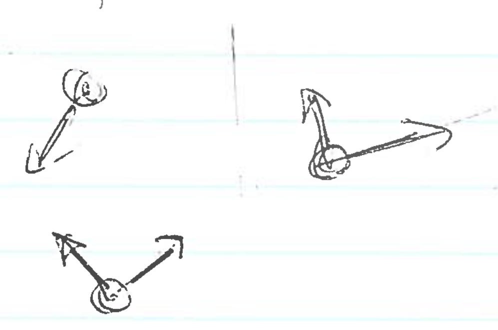
Amplitudes $\vec{E}_0$ $\vec{E}_0 \cdot \vec{k} = 0$ two possibilities.

"$\perp$" $\vec{E}_0 \perp$ scattering plane. $\vec{E}_0 = E_0\hat{y}$



"$\parallel$" E_0 in scattering plane $\Rightarrow$ $\vec{B}_0 \perp$ scattering.

$\vec{B}_0 = B_0\hat{y}$



$\mu\epsilon\frac{\partial \vec{E}}{\partial t} = \vec{\nabla}\times\vec{B}$

$-\frac{n^2}{c^2}\omega\vec{E} = \vec{k}\times\vec{B}$

(B)

(1) $\hat{z} \cdot \left(\epsilon(\vec{E}_0 + \vec{E}_0'') \hat{y} - \epsilon' E_0' \hat{y} \right) = 0$ ✓ ($\Delta D_\perp = 0$)

$\hat{z} \times (E_0 + E_0'' - E_0') \hat{y} = 0$ ($\Delta \vec{E}_\parallel = 0$)

$$\boxed{E_0 + E_0'' - E_0' = 0}$$

$\hat{n} \cdot (\vec{k} \times \vec{E}) = (\hat{n} \times \vec{k}) \cdot \vec{E} = (k \sin i \, \hat{y}) \cdot \vec{E}$ ($\Delta B_\perp = 0$)

$(k \sin i)(\hat{y} \cdot \vec{E}_0) + (k'' \sin r'')(\hat{y} \cdot \vec{E}_0'') = (k' \sin r')(\hat{y} \cdot \vec{E}_0')$

$$\boxed{E_0 + E_0'' - E_0' = 0}$$ (more luck)

($\Delta H_\parallel = 0$)

$\hat{n} \times (\vec{k} \times \vec{E}_0) = \vec{k}(\hat{n} \cdot \vec{E}_0) - \vec{E}_0 (\hat{n} \cdot \vec{k}) = -E_0 \hat{y} (\hat{n} \cdot \vec{k})$

$\hat{z} \cdot \hat{y}$

$\frac{1}{\mu} E_0 (k \cos i) + \frac{1}{\mu} E_0'' (-k \cos i) - \frac{1}{\mu'} E_0' (k' \cos r')$

($\Delta H_\parallel = 0$) ?

$$\boxed{\frac{n}{\mu} \cos i \, E_0 - \frac{n}{\mu} \cos i \, E_0'' - \frac{n'}{\mu'} \cos r' \, E_0' = 0}$$

$n'^2 \cos^2 r' = n'^2 (1 - \sin^2 r') = n'^2 - n'^2 \sin^2 r' = \underline{n'^2 - n^2 \sin^2 i}$

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I

$$\frac{E_0'}{E_0} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\cos^2 i}}$$

$$\frac{E_0''}{E_0} = \frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}$$

(7.39)

(II)

$$\frac{nE_0}{\mu} + \frac{nE_0''}{\mu} = \frac{n'E_0'}{\mu'}$$ $\Delta H_{\parallel} = 0$

$$E_0\cos i - E_0''\cos i = E_0'\cos r$$ $\Delta E_{\perp} = 0$

II

$$\frac{E_0'}{E_0} = \frac{2nn'\cos i}{\frac{\mu}{\mu'}n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$$

$$\frac{E_0''}{E_0} = \frac{\frac{\mu}{\mu'}n'^2\cos i - n\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$$

(7.41)