1/15/2016

7:39
7:41

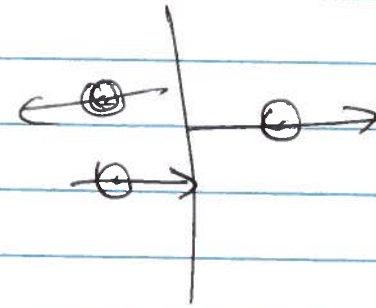
Often (usually) (always?) $\mu \approx \mu' = \mu_0$

Normal incidence, $\sin i \approx 0$ $(\cos i \approx 1)$

$\perp$

$$\frac{E_0'}{E_0} = \frac{2n}{n+n'}$$

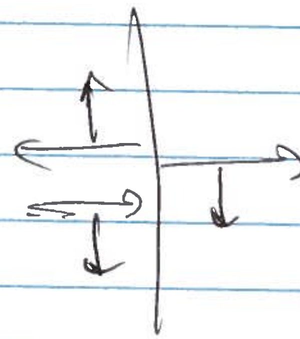
$$\frac{E_0''}{E_0} = \frac{n-n'}{n+n'}$$



$\parallel$

$$\frac{E_0'}{E_0} = \frac{2n}{n+n'}$$

$$\frac{E_0''}{E_0} = \frac{n'-n}{n+n'}$$



RHR

$n' > n$ phase-reversed

$n' \gg n$ $E_0'' \approx -E_0$. "ideal" mirror. (imaginary = more perfect)

reflection / transmission $\longleftrightarrow$ Power

Energy transport → $\vec{S}$

$$\langle \vec{S}\cdot\hat{n} \rangle = \mathrm{Re}\left[\frac{1}{2}\, \hat{n}\cdot\left(\vec{E}\times\vec{H}^*\right) \right]$$

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{k}\times\vec{E} = \omega\vec{B}$$

$$\hat{n}\cdot\left(\vec{E}\times\vec{H}^*\right) = \hat{n}\cdot\left(\vec{E}\times\left(\frac{1}{\mu\omega}\vec{k}\times\vec{E}\right)^*\right)$$

$$= \left(\frac{1}{\mu\omega}\right)^* \hat{n}\cdot\left(\vec{k}^*\left(\vec{E}\cdot\vec{E}^*\right) - \vec{E}^*\cancel{\left(\vec{k}^*\cdot\vec{E}\right)}\right)$$

$$= \left(\frac{k}{\mu\omega}\right)^* \hat{n}\cdot\hat{k}\,\left|\vec{E}_0\right|^2$$

$$\langle \vec{S}\cdot\hat{n} \rangle = \frac{1}{2}\,\mathrm{Re}\left(\frac{k}{\mu\omega}\right)\left|E_0\right|^2\ \hat{n}\cdot\hat{k}$$

Normal : $\underline{\hat{n}\cdot\hat{k} = 1}$ $\quad k = \frac{n\omega}{c} \quad k' = \frac{n'\omega}{c} \quad \mu = \mu' = \mu$

$$P_0 = P_i = \frac{1}{2}\,\mathrm{Re}\left(\frac{n\omega/c}{\mu\omega}\right) = \frac{1}{2\mu c}\,\mathrm{Re}\,n\,\left|E_0\right|^2$$

$$\boxed{P_i = \frac{1}{2}\,\mathrm{Re}\,n\cdot\frac{\left|E_0\right|^2}{Z_0}} \qquad \boxed{P' = \frac{1}{2}\,\mathrm{Re}\,n'\,\frac{\left|E'\right|^2}{Z_0}}$$

$$T = \frac{P'}{P_0} = \frac{n'\left|E_0'\right|^2}{n\left|E_0\right|^2} = \frac{n'\cdot 4n^2}{n\,(n+n')^2} = \frac{4nn'}{(n+n')^2}$$

$$R = \frac{P''}{P_0} = \frac{\cancel{n}\left|E_0''\right|^2}{\cancel{n}\left|E_0\right|^2} = \frac{(n-n')^2}{(n+n')^2} \qquad \boxed{R+T=1}$$

$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{\mu_0 c}$

details

(i) $$\frac{E_0''}{E_0} = \frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}$$

(ii) $$\frac{E_0''}{E_0} = \frac{\frac{\mu}{\mu'}n'^2\cos i - n\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$$

if $n > n'$, $\sin i$ too big $\rightarrow \sqrt{\text{negative \#}}$

$$k^2 = \frac{n^2\omega^2}{c^2} \qquad k'^2 = \frac{n'^2\omega^2}{c^2}$$

$$k'^2_z = k'^2 - k'^2_\parallel = k'^2 - k^2_\parallel$$

$$= \frac{n'^2\omega^2}{c^2} - \frac{n^2\omega^2}{c^2}\sin^2 i = \frac{\omega^2}{c^2}\left(n'^2 - n^2\sin^2 i\right)$$

$\rightarrow$ $\boxed{k'_z \text{ imaginary}}$ $\quad e^{i(\vec{k}'\cdot\vec{x})} \rightarrow e^{i\vec{k}_\parallel'\cdot\vec{x}_\parallel}\, \underline{\underline{e^{-\kappa z}}}$

$$\langle \vec{S}'\cdot\hat{n}\rangle = \ldots \mathrm{Re}(\hat{n}\cdot\vec{k}') \rightarrow \mathrm{Re}(k'_z) = 0$$

$$R = \frac{|E_0''|^2}{|E_0|^2} = \frac{|X - iY|^2}{|X + iY|^2} = \frac{|Z^*|^2}{|Z|^2} = \frac{|re^{-i\theta}|^2}{|re^{+i\theta}|^2} = 1$$

$n > n'$ → inside something

→ "Total internal reflection"

critical angle: $n'^2 - n^2 \sin^2 i = 0$

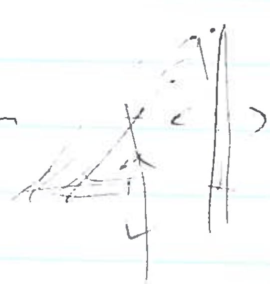
$$\boxed{\sin i_c = \frac{n'}{n}}$$ needs $n > n'$

$\sin \theta' = 1$

all of outside world → circle θ_c.
beyond → silvery

Crown glass (K5) $n = 1.522$
Dense flint (SF6) $n = 1.805$

$n > \sqrt{2}$ isosceles prism



Binoculars
periscope

(5)

(∥ only) $E_0^\parallel \neq 0$ $\quad \sqrt{\frac{\mu}{\mu'}}\, n' \cos i = n\sqrt{n'^2 - n^2 \sin^2 i}$

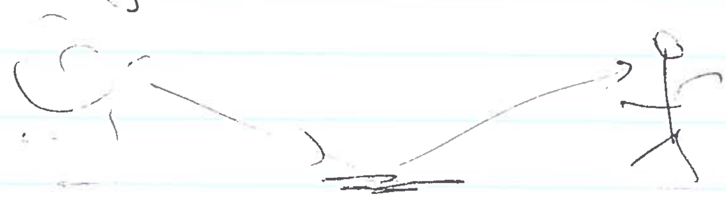
$$n'^2 \cos^2 i = n^2 n'^2 - n^4 \sin^2 i$$

$$\frac{n'^2 \cos^2 i}{n^2} + \frac{n^2 \sin^2 i}{n'^2} = 1$$

$$\cos^2 i = \frac{n^2}{n^2+n'^2} \qquad \sin^2 i = \frac{n'^2}{n^2+n'^2}$$

Brewster's angle $\quad \tan i_B = \frac{n'}{n}$

Always has a solution



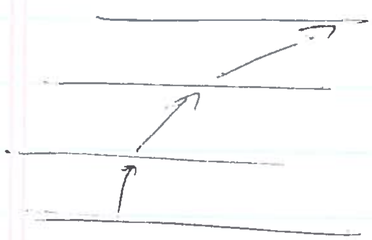
puddle, chrome bumper, mirror, ...

polarized filter remove ∥
polarized sunglasses allow ⊥

(⊥) $E_0^\parallel \neq 0 \to n\cos i = \sqrt{n'^2 - n^2\sin^2 i}$

$$n^2\cos^2 i + n^2 \sin^2 i = n'^2 \qquad (n = n')$$

$n = 1$, $n' = 1.3$ $\quad \tan i_B = 1.3 \quad i_B \approx 52^\circ$



layers $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 \ldots$

Smoothly varying $\varepsilon(\vec{x})$

§8.10

$$\vec\nabla\cdot\vec D = \vec\nabla\cdot(\varepsilon\vec E) = \varepsilon(\vec\nabla\cdot\vec E) + \vec E\cdot(\vec\nabla\varepsilon) = 0$$

$$\boxed{\vec\nabla\cdot\vec E = -\frac{1}{\varepsilon}(\vec\nabla\varepsilon)\cdot\vec E}$$ looks like a (field-dependent!) source

maxwell → $\vec\nabla\times(\vec\nabla\times\vec E) = \vec\nabla(\vec\nabla\cdot\vec E) - \nabla^2\vec E$

$$= \vec\nabla\times\left(-\frac{\partial\vec B}{\partial t}\right) = \vec\nabla\times(+i\mu\omega\vec H) = i\mu\omega(-i\omega\varepsilon\vec E)$$

$$\nabla^2\vec E + \mu\varepsilon\omega^2\vec E = -\vec\nabla\left(\frac{1}{\varepsilon}(\vec\nabla\varepsilon)\cdot\vec E\right) \quad (8.108)$$

$$\nabla^2\vec E + \frac{n^2\omega^2}{c^2}\vec E = -\vec\nabla\left(\frac{1}{n^2}(\vec\nabla n^2)\cdot\vec E\right) \quad (8.125)$$

Geometric optics limit: $\frac{n\omega}{c} \gg \vec\nabla$ $(\lambda \ll L)$

$\psi = e^{i\frac{\omega}{c}S}$ ψ = any component of $\vec E, \vec H, \Phi, \vec A, \ldots$

S = "eikonal" (OPD) optical path of a ray.

"IKON"