1/20/2016

$\varepsilon = \varepsilon(\vec{x})$, $e^{-i\omega t}$

$$\nabla^2\vec{E} + \mu\varepsilon\omega^2\vec{E} = -\vec{\nabla}\left(\frac{1}{\varepsilon}(\vec{\nabla}\varepsilon)\cdot\vec{E}\right)$$

$$\boxed{\nabla^2\vec{E} + \frac{n^2\omega^2}{c^2}\vec{E} = -\vec{\nabla}\left(\frac{1}{n^2}(\vec{\nabla}n^2)\cdot\vec{E}\right)} \quad (8.125)$$

in general → general eqs.

<u>geometric optics</u> limit ($\lambda \ll L$)

$\left(\frac{n\omega}{c} \gg \vec{\nabla}\right)$

$\psi = e^{\frac{i\omega}{c}S}$ ψ = any $\vec{E}, \vec{B}, \Phi, \vec{A}, \ldots$

S = "eikonal" (OED: optical path of a ray. "IKON")

$$\vec{\nabla}\psi = \frac{i\omega}{c}\vec{\nabla}S\cdot\psi$$

$$\nabla^2\psi = -\frac{\omega^2}{c^2}(\vec{\nabla}S)^2\psi + i\frac{\omega}{c}(\nabla^2 S)\psi.$$

$$\vec{E} = \vec{e}\,\psi.$$

$$\nabla^2\vec{E} + \frac{n^2\omega^2}{c^2}\vec{E} = -\frac{\omega^2}{c^2}(\vec{\nabla}S)^2\vec{E} + i\frac{\omega}{c}(\nabla^2 S)\vec{E} + \frac{n^2\omega^2}{c^2}\vec{E}$$

$$+ \left[\frac{2i\omega}{c}(\vec{\nabla}S\cdot\vec{\nabla})\vec{e}\right]\psi + \psi(\nabla^2\vec{e})$$

②

$\frac{n\omega}{c} \gg \vec{\nabla} \sim \frac{1}{L}$ $\boxed{\vec{\nabla}S \cdot \vec{\nabla}S = n^2_{(\vec{r})}}$ (8.111)

$[S] =$ length. $\vec{\nabla}S$ dimensionless.

near $\vec{x} = 0$: $S \simeq S_0 + (\vec{\nabla}S)_0 \cdot \vec{x} \ldots$

$$\psi \sim e^{i\frac{\omega}{c}S} = \left(e^{i\frac{\omega}{c}S_0}\right) \underbrace{e^{i\frac{\omega}{c}(\vec{\nabla}S)_0 \cdot \vec{x}}}_{e^{i\vec{k}\cdot\vec{x}}}$$

$$\boxed{\vec{k} = \frac{\omega}{c}\vec{\nabla}S}$$ locally.

Ray tracing: take a step in direction $\hat{k}$

$\vec{\Delta r} = \hat{k}\,\Delta s$ $\vec{r}$: vector displacement

Δs - path length

$\Delta s \to 0$ $\boxed{\frac{d\vec{r}}{ds} = \hat{k}}$

$$\boxed{n\frac{d\vec{r}}{ds} = n\hat{k} = \vec{\nabla}S}$$ (8.113)

$$\frac{d}{ds}\left(n\frac{d\vec r}{ds}\right) = \frac{d}{ds}\left(\vec\nabla S\right) = \vec\nabla\left(\frac{dS}{ds}\right)$$

$$\frac{d}{ds}(\text{anything}) = \frac{d}{ds}(\text{direction } \hat k) = \hat k\cdot\vec\nabla$$

$$\frac{dS}{ds} = (\hat k\cdot\vec\nabla)S = \hat k\cdot(\hat k n) = n$$

$$\boxed{\frac{d}{ds}\left(n\frac{d\vec r}{ds}\right) = \vec\nabla\left[n(\vec r(s))\right]} \quad (8.114)$$

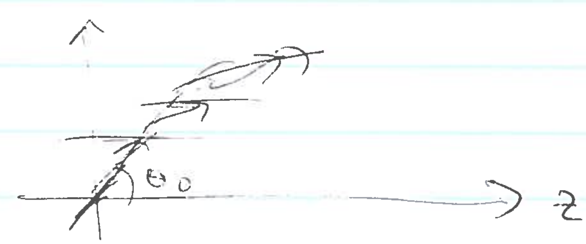
All about the path (no S). ("$F = ma$")

Let $\frac{\partial n}{\partial z} = 0$ (1D. $n = n(x)$) ($n = n(\ell)$)

$$\hat k = \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = \frac{(dx^2+dy^2+dz^2)}{ds^2} = \left(\frac{ds}{ds}\right)^2 = 1$$

choose $\phi = 0$.



$$\frac{d}{ds}\left[n(x)\sin\theta\right] = \frac{dn}{dx}$$

$$\frac{d}{ds}\left(n(x)\cos\theta\right) = 0$$

$\hookrightarrow n(x)\cos\theta = \text{constant} = n_0\cos\theta_0 \equiv \bar n$

$$\boxed{n(x)\cos\theta(x) = n_0\cos\theta_0}$$ Snell's Law.

From $x(s)$, $z(s)$ $\rightarrow$ write $\boxed{x = x(z)}$

$$\frac{df}{ds} = \frac{df}{dz}\frac{dz}{ds} = (\cos\theta)\frac{df}{dz} = \frac{\bar{n}}{n}\frac{df}{dz}.$$

$$\left(\frac{\bar{n}}{n}\frac{d}{dz}\right)\left[\cancel{n}\,\frac{\bar{n}}{\cancel{n}}\frac{dx}{dz}\right] = \frac{dn}{dx} \qquad (8.116)$$

$$\boxed{\bar{n}^2\frac{d^2x}{dz^2} = n\frac{dn}{dx} = \frac{d}{dx}\left(\frac{1}{2}n^2\right)}$$

"$F = ma$"

$V = -\frac{1}{2}n^2$

1st Integral
$$\bar{n}^2\frac{d^2x}{dz^2}\frac{dx}{dz} = \frac{d}{dz}\left(\frac{1}{2}\bar{n}^2\left(\frac{dx}{dz}\right)^2\right)$$

$$\frac{d}{dx}\left(\frac{1}{2}n^2\right)\frac{dx}{dz} = \frac{d}{dz}\left(\frac{1}{2}n^2\right)$$

$$\frac{d}{dz}\left[\frac{1}{2}\bar{n}^2\left(\frac{dx}{dz}\right)^2 - \frac{1}{2}n^2\right] = 0$$

$$\frac{1}{2}\bar{n}^2\left(\frac{dx}{dz}\right)^2 - \frac{1}{2}n^2 = \text{constant} = -\frac{1}{2}\bar{n}^2$$

$$\left(\frac{dx}{dz}\right) = 0 \quad \sin\theta = 0 \mid \cos\theta = 1, \quad n = \bar{n}$$

$$\boxed{\frac{1}{2}\bar{n}^2\left(\frac{dx}{dz}\right)^2 - \frac{1}{2}n^2 = -\frac{1}{2}\bar{n}^2}$$

"Energy Conservation"

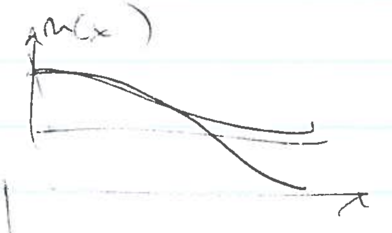
(8.117)

JEF [signature]

(5)

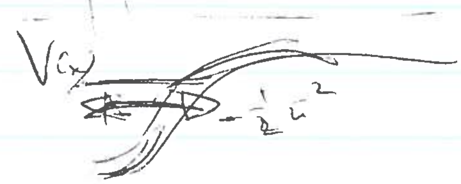
"confinement" (graded fiber).

monotonic decreasing $n(x)$



$\leftrightarrow$ confining potential

$$V = -\frac{1}{2}n^2$$



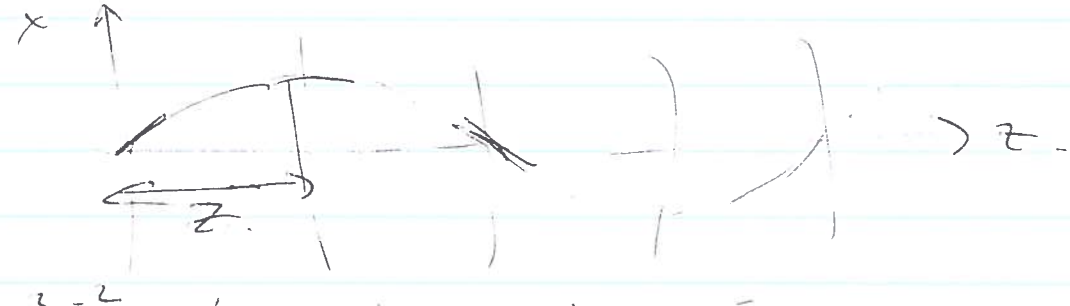
Angle low enough $\leftrightarrow$ confined.

$$\frac{1}{2}\bar{n}^2\left(\frac{dx}{dz}\right)^2 = -\frac{1}{2}\bar{n}^2 + \frac{1}{2}n^2$$

$\int x_{max}$: $\frac{dx}{dz} = 0$. $n = \bar{n}$. $= n_0 \cos\theta_0$

$\sin\theta = 0$ $\cos\theta = 1$. $n = \bar{n}$

1-d. confined motion (periodic)



$\left(\frac{dx}{dz}\right)^2 = \frac{n^2 - \bar{n}^2}{\bar{n}^2}$ (energy) $\rightarrow$ $\frac{dx}{dz} = \frac{\bar{n}}{\sqrt{n^2 - \bar{n}^2}}$

$$\int_0^Z dz = Z = \int_0^{x_{max}} \frac{dz}{dx}\,dx = \int_0^{x_{max}} \frac{\bar{n}\,dx}{\sqrt{n^2 - \bar{n}^2}} \quad (2)$$