1/29/2016

$$-\omega^2 \vec{v}_1 + (c_s^2 + v_A^2)\vec{k}(\vec{k}\cdot\vec{v}_1) + (\vec{k}\cdot\vec{v}_A)^2 \vec{v}_1 - \vec{k}(\vec{k}\cdot\vec{v}_A)(\vec{v}_A\cdot\vec{v}_1) - \vec{v}_A(\vec{k}\cdot\vec{v}_A)(\vec{k}\cdot\vec{v}_1) = 0$$

(7.78)

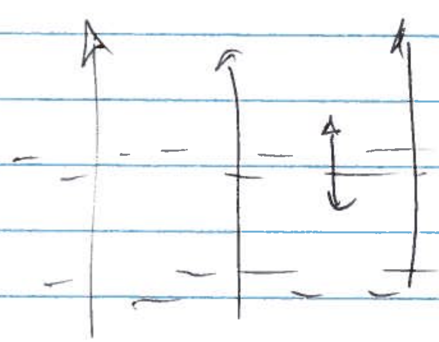
II. $\vec{k} \parallel \vec{B}_0$ $\quad$ B $\parallel \vec{v}_1$ also.

$$-\omega^2 + (c_s^2 \cancel{+ v_A^2})k^2 \cancel{+ k^2 v_A^2} - \cancel{k^2 v_A^2} - \cancel{k^2 v_A^2} = 0$$

$$\boxed{\omega^2 = c_s^2 k^2}$$

$$-i\omega\rho_1 + \rho_0\, i\vec{k}\cdot\vec{v}_1 = 0 \qquad \boxed{\rho_1 = \frac{v_1}{c_s}\rho_0}$$

$$-i\omega\vec{B}_1 = i\vec{k}\times(\vec{v}_1\times\vec{B}_0) = 0 \qquad \boxed{\vec{B}_1 = 0}$$



motion $\parallel \vec{B}$

no magnetic force

<u>sound wave</u>

one last MHD thing: dissipation $\underline{\sigma, \eta}$

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}p + \frac{1}{\mu}(\vec{\nabla}\times\vec{B})\times\vec{B}_0$$

viscosity

$$+\, \eta\, \nabla_i \Big[\underbrace{\nabla_j v_i + \nabla_i v_j - \tfrac{2}{3}\delta_{ij}(\vec{\nabla}\cdot\vec{v})}_{\eta \nabla^2 \vec{v}}\Big]$$

Alfvén wave (example)

$\vec{v}_1 \perp \vec{k} \Leftrightarrow \vec{\nabla}\cdot\vec{v} = 0$

$$\rho_0 \frac{\partial}{\partial t}(\delta\vec{v}) = -c_s^2 \vec{\nabla}(\delta\rho) - \frac{1}{\mu}\vec{B}_0\times(\vec{\nabla}\times\delta\vec{B}) + \eta\nabla^2\delta\vec{v}$$

conductivity

$$\frac{\partial}{\partial t}(\delta\vec{B}) = \vec{\nabla}\times(\delta\vec{v}\times\vec{B}_0) + \frac{1}{\mu\sigma}\nabla^2(\delta\vec{B})$$

$$\frac{\partial}{\partial t}(\delta\rho) + \rho_0 \cancel{\vec{\nabla}\cdot(\delta\vec{v})} = 0 \qquad \underline{\delta\rho = 0} \qquad (c_s^2 \text{ disappearing})$$

$$-\rho_0 i\omega \vec{v}_1 = -\frac{1}{\mu}\vec{B}_0\times(i\vec{k}\times\vec{B}_1) + \eta k^2 \vec{v}_1$$

$$\vec{k}(\cancel{\vec{B}_0\cdot\vec{B}_1}) - \vec{B}_1(\vec{k}\cdot\vec{B}_0)$$

$$-\cancel{i}\omega\rho_0\left[1 + \frac{i\eta k^2}{\omega\rho_0}\right]\vec{v}_1 = \cancel{i}\,\frac{\vec{B}_1}{\mu}(\vec{k}\cdot\vec{B}_0)$$

③

$$-i\omega \vec{B}_1 = i\vec{k}\times(\vec{v}_1\times\vec{B}_0) - \frac{k^2}{\mu\sigma}\vec{B}_1$$

$$\vec{v}_1(\vec{k}\cdot\vec{B}_0) - \vec{B}_0(\cancel{\vec{k}\cdot\vec{v}_1})$$

$$\boxed{-i\omega\left(1+\frac{ik^2}{\mu\sigma\omega}\right)\vec{B}_1 = i\vec{v}_1(\vec{k}\cdot\vec{B}_0)}$$

$$\omega\left(1+\frac{ik^2}{\mu\sigma\omega}\right)\cdot\omega\rho_0\left(1+\frac{i\eta k^2}{\omega\rho_0}\right)\vec{v}_1$$

$$= \frac{1}{\mu}\cdot\omega\left(1+\frac{ik^2}{\mu\sigma\omega}\right)\vec{B}_1(\vec{k}\cdot\vec{B}_0)$$

$$= \frac{1}{\mu}\cdot(\vec{k}\cdot\vec{B}_0)^2\,\vec{v}_1$$

$$\omega^2\rho_0\left(1+\frac{i\eta k^2}{\omega\rho_0}\right)\left(1+\frac{ik^2}{\mu\sigma\omega}\right) = \frac{k^2B_0^2}{\mu}$$

$$\boxed{\omega^2\left(1+\frac{i\eta k^2}{\omega\rho_0}\right)\left(1+\frac{ik^2}{\mu\sigma\omega}\right) = k^2v_A^2}$$

(4)

$$k \simeq \frac{\omega}{v_A}\left(1 + \frac{1}{2}\frac{i\eta}{\omega\rho_0}\frac{\omega^2}{v_A^2} + \frac{1}{2}\frac{i}{\mu\sigma\omega}\frac{\omega^2}{v_A^2}\right) + \cdots$$

$$\boxed{k \simeq \frac{\omega}{v_A} + \frac{1}{2} i \frac{\omega^2}{v_A^2}\left(\frac{\eta}{\rho_0} + \frac{1}{\mu\sigma}\right)}$$

either $\sigma, \eta \to$ Im k $\to e^{-kz}$,

Higher freq., charge separation, $\frac{\partial \vec{E}}{\partial t}$.

too many ρ's — Let $n(\vec{x}, t)$ electrons

no nuclei (ions)

$$\boxed{\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) = 0}$$

$\vec{B}_0 = 0$

$$mn\left(\frac{\partial \vec{v}}{\partial t} + \cancel{(\vec{v}\cdot\vec{\nabla})\vec{v}}\right) = -\vec{\nabla}p + \left(ne\vec{E} + \cancel{ne\vec{v}\times\vec{B}}\right)$$

$$\boxed{\frac{\partial \vec{v}}{\partial t} = \frac{e}{m}\vec{E} - \frac{1}{nm}\vec{\nabla}p}$$

$\hookrightarrow \frac{1}{mn_0}\left(\frac{\partial p}{\partial n}\right)_0 \vec{\nabla}n$

$$\vec{\nabla}\cdot\vec{E} = e(n-n_0)/\varepsilon$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 n e\vec{v} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial^2 \overset{\Delta n}{n}}{\partial t^2} + n_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{v}) = 0$$

$$\vec{\nabla}\cdot\left(\frac{\partial\vec{v}}{\partial t}\right) = \frac{e}{m}\underbrace{\vec{\nabla}\cdot\vec{E}}_{\frac{e\,\Delta n}{\varepsilon}} - \frac{1}{mn_0}\left(\frac{\partial p}{\partial n}\right)_0 \nabla^2(\Delta n)$$

$$\boxed{\frac{\partial^2}{\partial t^2}(\Delta n) + \underbrace{\frac{e^2 n_0}{m\varepsilon_0}}_{\omega_p^2}\Delta n - \frac{1}{m}\left(\frac{\partial p}{\partial n}\right)_0\nabla^2(\Delta n) = 0}$$

$$\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = \mu_0 n_0 e\frac{\partial\vec{v}}{\partial t} + \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$= \vec{\nabla}(\vec{\nabla}\cdot\vec{E})$$

$$= \mu_0 n_0 e\left(\frac{e}{m}\vec{E} - \frac{1}{mn_0}\left(\frac{\partial p}{\partial n}\right)_0 \vec{\nabla}n\right)$$

$$\boxed{\frac{\partial^2\vec{E}}{\partial t^2} + \frac{n_0 e^2\vec{E}}{m\varepsilon_0} - \frac{1}{m}\left(\frac{\partial p}{\partial n}\right)_0\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = c^2\vec{\nabla}\times\cancel{\frac{\partial\vec{B}}{\partial t}}}$$

$\vec{B}=0$ $[\vec{\nabla}\times\vec{E}=0$ $\vec{E}=-\vec{\nabla}\Phi$ longitudinal

$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) \to \nabla^2(\vec{\nabla}\Phi) = \nabla^2\vec{E}$

$$-\omega^2 + \omega_p^2 + c_s^2 k^2 = 0$$

$$\boxed{\omega^2 = \omega_p^2 + c_s^2 k^2}$$ longitudinal $\vec{E}$

$$\boxed{\omega^2 = \omega_p^2 + c^2 k^2}$$ transverse $\vec{E}$

[source elsewhere]

uncertainty

~~Causality.~~
~~Kramers - Kronig.~~
~~Complex Contour integrals~~

$\boxed{\text{Chapter 9}}$ Radiating Systems

oscillating sources $\rho, \vec{J} \sim e^{-i\omega t}$ (Re)

Lorentz gauge $\boxed{\frac{1}{c^2}\frac{\partial \Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0}$ (μ_0, ϵ_0)

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi = -\rho/\epsilon_0$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\mu_0\vec{J}$$

$$\longrightarrow \boxed{\begin{aligned}(\nabla^2 + k^2)\Phi &= -\rho/\epsilon_0 \\ (\nabla^2 + k^2)\vec{A} &= -\mu_0\vec{J}\end{aligned}}$$

"Helmholtz Equation"

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi = \frac{\hbar^2 k^2}{2m}\psi$$

$$\longrightarrow \boxed{(\nabla^2 + k^2)\psi = \frac{2mV}{\hbar^2}\psi}$$