2|5|2016

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int d^3x' \left[\vec{J}(\vec{x}') + \frac{1}{2}\left[\vec{J}(\hat{r}\cdot\vec{x}') - \vec{x}'(\hat{r}\cdot\vec{J}) \right] + \frac{1}{2}\left(\vec{J}(\hat{r}\cdot\vec{x}') + \vec{x}'(\hat{r}\cdot\vec{J}) \right) \right] \quad -ik$$

$$\hat{r}_j \left(x'_i J_j + x'_j J_i \right)$$

$$\nabla'_k \left(x'_i x'_j J_k \right) = \delta_{ik} x'_j J_k + x'_i \delta_{jk} J_k + x'_i x'_j \nabla_k J_k$$

$$= x'_i J_j + x'_j J_i + i\omega\rho x'_i x'_j$$

$$\int d^3x' \nabla'_k (\;) \rightarrow \oint_{R \to \infty} d^2a' \, \hat{n}'_k (\;) \rightarrow 0$$

$$\int d^3x' \left(x'_i J_j + x'_j J_i + i\omega\rho x'_i x'_j \right) = 0.$$

②

$$\vec{A} = -ck^2 \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \frac{1}{2} \hat{r}_j \int d^3x' \, x'_i x'_j \rho$$

recall. $Q_{ij} = \int d^3x' \left(3x'_i x'_j - \delta_{ij} r'^2\right)$

gauge

$$\vec{A} = -ck^2 \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \frac{1}{6} \hat{r}_j \left(Q_{ij} + C \cdot \delta_{ij}\right)$$

$$\vec{A} = -\frac{ck^2 \mu_0}{24\pi} \frac{e^{ikr}}{r} \left[\vec{Q}(\hat{r}) + C\hat{r}\right]$$

$\vec{Q}(\hat{r}) = Q_{ij} \hat{r}_j$ (9.38)

$$\vec{H} = \frac{1}{\mu_0} ik\hat{r} \times \vec{A}$$

$$\vec{H} = \frac{-ick^3}{24\pi} \frac{e^{ikr}}{r} \hat{r} \times \vec{Q}(\hat{r})$$

$\varepsilon_{ijk} \hat{r}_j Q_{km} \hat{r}_m$

$$\vec{E} = Z_0 \vec{H} \times \hat{r}$$

$$\frac{dP}{d\Omega} = \frac{1}{2} r^2 \hat{r} \cdot \left(\vec{E} \times \vec{H}^*\right)$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^6}{1152\pi^2} \left| \left(\hat{r} \times \vec{Q}(\hat{r})\right) \times \hat{r} \right|^2$$

$2(24)^2$.

<u>Case</u>: rotationally symmetric about z-axis.

$$Q_{ij} = \int d^3x' \,(3x'_i x'_j - \delta_{ij} r'^2)\,\rho$$

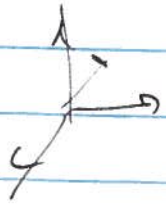
xy xz yz
12, 13, 23 $\rightarrow$ ($\sin\phi' \cos\phi'$), ($\sin\phi'$), ($\cos\phi'$) $\longrightarrow = 0$

(xx) (yy) $\int d^3x' \,(3r'^2 \sin^2\theta' \cos^2\phi' - r'^2)\,\rho \quad = -\frac{1}{2} Q_0$

$$\frac{3}{2}\sin^2\theta' - 1 = \frac{3}{2}(1-\cos^2\theta) - 1 = \frac{1}{2} - \frac{3}{2}\cos^2\theta$$

(zz) $\int d^3x' \,(3r'^2 \cos^2\theta' - r'^2)\,\rho \quad + Q_0$

$$Q_{ij} = \begin{pmatrix} -\frac{1}{2}Q_0 & & \\ & -\frac{1}{2}Q_0 & \\ & & Q_0 \end{pmatrix}$$



$$\vec{Q}(\hat{r}) = Q_0\begin{pmatrix} -\frac{1}{2}\sin\theta\cos\phi\,\hat{x} \\ -\frac{1}{2}\sin\theta\sin\phi\,\hat{y} \\ +\cos\theta\,\hat{z} \end{pmatrix} = Q_0\left(-\frac{1}{2}\hat{r} + \frac{3}{2}\cos\theta\,\hat{z}\right)$$

$\underline{\hat{r}\times\vec{Q}} : \frac{3}{2} Q_0 \cos\theta \,(\sin\theta)(-\hat{\phi}) \quad \leftarrow \hat{B}$

$(\hat{r}\times\vec{Q})\times\hat{r} = \left(\frac{3}{2}Q_0 \sin\theta\cos\theta\right)(-\hat{\theta}) \quad \leftarrow \hat{t}$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^6}{1152\pi^2}\cdot\frac{9}{4}|Q_0|^2 \sin^2\theta\cos^2\theta \qquad (9.51)$$

$\frac{1}{512\pi^2}$

Total power

$$|(\hat{r}\times\vec{Q})\times\hat{r}|^2 = |\hat{r}\times\vec{Q}|^2$$

$$= (\hat{r}\times\vec{Q})\cdot(\hat{r}\times\vec{Q})^* = \hat{r}\cdot[\vec{Q}\times(\hat{r}\times\vec{Q}^*)]$$

$$= \vec{Q}\cdot\vec{Q}^* - (\hat{r}\cdot\vec{Q})(\hat{r}\cdot\vec{Q}^*)$$

$$= Q_{ij}\hat{r}_j Q^*_{ik}\hat{r}_k - Q_{ij}\hat{r}_i\hat{r}_j Q^*_{mn}\hat{r}_m\hat{r}_n .$$

$$\int d\Omega\, \hat{r}_i\hat{r}_j = \frac{4\pi}{3}\delta_{ij}$$

$$\int d\Omega\, \hat{r}_i\hat{r}_j\hat{r}_m\hat{r}_n = K\cdot(\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})$$

$i=j=m=n=3$

$$\int d\Omega\,(\cos^4\theta) = 2\pi\int_{-1}^{1} d\mu\cdot\mu^4 = \frac{4\pi}{5} = K(1+1+1)$$

$$\int d\Omega\, \hat{r}_i\hat{r}_j\hat{r}_m\hat{r}_n = \frac{4\pi}{15}(\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})$$

$$\int d\Omega\, |\hat{r}\times\vec{Q}|^2 = Q_{ij}Q^*_{ik}\frac{4\pi}{3}\delta_{jk}$$

$$- Q_{ij}Q^*_{mn}\frac{4\pi}{15}(\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})$$

$$= Q_{ij}Q^*_{ij}\left(\frac{4\pi}{3} - \frac{4\pi}{15}\cdot 2\right)$$

$$\frac{4\pi}{15}(5-2) = \frac{4\pi}{15}\cdot 3 = \frac{4\pi}{5}.$$

5

$$P = \frac{c^2 Z_0 k^6}{1152\pi^2} \cdot \frac{4\pi}{5} Q_{ij} Q_{ij}^* = \frac{c^2 Z_0}{1440\pi} k^6 |Q|^2$$

Gravity $\quad c Z_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{\epsilon_0}$.

$$P = \frac{1}{360} \cdot \frac{1}{4\pi\epsilon_0} c k^6 \cdot |Q_{ij}|^2$$

analog. $\frac{1}{4\pi\epsilon_0} q^2 \to G m^2$.

$$Q_{ij} = \int d^3x \left(3x_i x_j - \delta_{ij} r^2\right) \rho_e$$

$$I_{ij} = \int d^3x \left(x_i x_j - \tfrac{1}{3} \delta_{ij} r^2\right) \rho_m.$$

~~$P = \frac{1}{360} c k^6 \cdot$~~

$$P = \frac{1}{40} c k^6 \cdot \frac{1}{4\pi\epsilon_0} \left|\frac{Q_{ij}}{3}\right|^2 \to \frac{1}{40} c k^6 G |I_{ij}|^2$$

MTW (36.1)

$$P_{GW} = \frac{1}{10} \cdot c k^6 G |I_{ij}|^2$$

spin 2 vs spin 1 $\qquad$ factor of 4