2/10/2016

$$\frac{dP}{d\Omega} = \frac{Z_0 |I_0|^2}{8\pi^2}\left[\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta}\right]^2$$

$kd = \pi$

$$\frac{Z_0 |I_0|^2}{8\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$kd = 2\pi$

$$\frac{Z_0 |I_0|^2}{8\pi^2} \, 4 \, \frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}.$$

dipole

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0 \qquad \boxed{\lambda = \frac{1}{i\omega}\frac{\partial I}{\partial z}}$$

$$\lambda = \frac{1}{i\omega} \cdot \frac{\partial}{\partial z} \cdot I_0 \sin\left[k\left(\frac{d}{2} - |z|\right)\right]$$

$$= \mp \frac{1}{i\omega} I_0 \, k \cdot \cos\left[k\left(\frac{d}{2} - |z|\right)\right]$$

$$\vec{p} = \int d^3x' \, \rho \, \hat{x} \rightarrow \int dz' \cdot \lambda \, \hat{z} \, z$$

$$= \frac{i I_0}{c} \hat{z} \int_{-d/2}^{d/2} dz' \, |z'| \cos\left(\frac{kd}{2} - k|z'|\right)$$

$$\boxed{\vec{p} = \frac{4 i I_0}{c} \hat{z} \, \frac{\sin^2\left(\frac{kd}{4}\right)}{k^2}} \rightarrow \frac{i I_0 d^2 \hat{z}}{4c}$$

②

Small d

$$P = \frac{8\pi}{3} \cdot \frac{1}{32\pi^2} \cdot c^2 Z_0 k^4 \cdot |\vec{p}|^2$$

$$\boxed{P = \frac{1}{192\pi} \cdot Z_0 I_0^2 \, (kd)^4}$$

$P = \frac{1}{2} |I_0|^2 R_{rad}$, $\quad \boxed{R_{rad} = \frac{1}{96\pi} Z_0 (kd)^4}$

$kd = \pi \quad \vec{p} = \frac{2i I_0}{ck^2} \qquad \frac{Z_0 |I_0|^2}{3\pi} \qquad R_{rad} = Z_0 \cdot \frac{2}{3\pi}$ $\quad \frac{\pi^3}{96} = \cancel{1.0704}$ 0.332 $\quad 0.212$

$kd = 2\pi \quad \vec{p} = \frac{4i I_0}{ck^2} \qquad R_{rad} = \frac{8}{3\pi} Z_0 \quad = 0.848 \qquad \frac{16\pi^3}{96} = \frac{\pi^3}{6} = 5.16$

3

$$P = \frac{1}{8\pi^2} Z_0 |I_0|^2 \int d\phi\, d\mu \frac{\left[\cos\left(\frac{kd}{2}\mu\right) - \cos\frac{kd}{2}\right]^2}{1-\mu^2}$$

$$= \frac{1}{2} Z_0 |I_0|^2 \times \frac{1}{2\pi} \Big[\gamma_E + \frac{1}{2}\gamma_E \cos kd + \log kd$$

$$+ \frac{1}{2} \cos kd \log\left(\frac{kd}{2}\right) \quad - - - - \Big]$$

Dipole not enough   multipole expansion

$(\nabla^2 + k^2)\psi = -j = 0$ (outside source)

$\psi(r,\theta,\phi,t) = R(r)\, Y_{lm}(\theta,\phi)\, e^{-i\omega t}$.

$$\longrightarrow \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + \frac{\omega^2}{c^2}\right) R = 0$$

Almost Bessel's equation. $\left(\frac{2}{r}\right)$ $(l(l+1))$

Behaviors: $k = \frac{\omega}{c}$ (kr big) $R'' + k^2 R = 0$ $R \sim \sin kr, \cos kr$

[and powers $\Rightarrow$ $\frac{\sin kr}{kr}$, $\frac{\cos kr}{kr}$]

kr small $R'' + \frac{2}{r}R' - \frac{l(l+1)}{r^2} = 0$ $R \sim r^l$, $1/r^{l+1}$.

[No γ_E, no "$\ln x$"]

let $R = u/\sqrt{r}$

$$R' = \frac{u'}{r^{1/2}} - \frac{1}{2}\frac{u}{r^{3/2}}$$

$$R'' = \frac{u''}{r^{1/2}} - 2\cdot\frac{1}{2}\cdot\frac{u'}{r^{3/2}} + \left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\frac{u}{r^{5/2}}$$

$$= \frac{u''}{r^{1/2}} - \frac{u'}{r^{3/2}} + \frac{3}{4}\frac{u}{r^{5/2}}.$$

$$\left(\frac{u''}{r^{1/2}} - \frac{u'}{r^{3/2}} + \frac{3}{4}\frac{u}{r^{5/2}}\right)$$

$$+ \frac{2}{r}\left(\frac{u'}{r^{1/2}} - \frac{1}{2}\frac{u}{r^{3/2}}\right) + \left(k^2 - \frac{l(l+1)}{r^2}\right)\frac{u}{r^{1/2}} = 0$$

$$\frac{1}{r^{1/2}}\left(u'' + u'\left(\frac{2}{r} - \frac{1}{r}\right) + \frac{u}{r^2}\left(\frac{3}{4} - 1\right) + \left(k^2 - \frac{l(l+1)}{r^2}\right)u\right) = 0$$

$$\boxed{u'' + \frac{1}{r}u' + \left(k^2 - \frac{(l+\frac{1}{2})^2}{r^2}\right)u = 0}$$

$$u = J_{l+\frac{1}{2}}(kr) \quad , \quad N_{l+\frac{1}{2}}(kr)$$

<u>let</u>:

$$\boxed{\begin{aligned} j_l(x) &= \sqrt{\frac{\pi}{2x}}\, J_{l+\frac{1}{2}}(x) \\ n_l(x) &= \sqrt{\frac{\pi}{2x}}\, N_{l+\frac{1}{2}}(x) \end{aligned}}$$

Spherical Bessel Functions

$$R = j_l(kr) \, , \, n_l(kr)$$