2/24/2010 Example. "small" dielectric sphere. $\mu = \mu_0$

$\Leftrightarrow$ $ka \ll 1$. $\varepsilon = K\varepsilon_0$.

$$\vec{E}_{inside} = \frac{3}{K+2}\vec{E}_0 \left(e^{-i\omega t}\right) \quad \left[\text{Ch. 4.} \ \langle \vec{E} \rangle_R = -\frac{1}{3\varepsilon_0}\vec{P} + \vec{E}_0\right]$$

$$\vec{P} = \varepsilon_0 \chi \vec{E}_{in} = \varepsilon_0 (K-1)\left(\frac{3\vec{E}_0}{K+2}\right)$$

$$\vec{p} = \frac{4\pi}{3}a^3 \vec{P} = 4\pi\varepsilon_0 \left(\frac{K-1}{K+2}\right)\vec{E}_0 a^3$$

$$\left(\frac{dP}{d\Omega}\right)_{in} = \frac{|E_0|^2}{2Z_0}$$

$$\left(\frac{dP}{d\Omega}\right)_{out} = \frac{c^2 Z_0}{32\pi^2} k^4 \left|(\hat{r}\times\vec{p})\times\hat{r}\right|^2 \qquad (9.22)$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{c^2 Z_0 k^4}{32\pi^2}\left|\frac{K-1}{K+2}\right|^2 a^6 (4\pi\varepsilon_0)^2 \left|\hat{r}\times\vec{E}_0\right|^2}{\frac{1}{2Z_0}|\vec{E}_0|^2}$$

$$= c^2 Z_0^2 \varepsilon_0^2 \frac{4\cdot(4\pi)^2}{32\pi^2} k^4 a^6 \left|\frac{K-1}{K+2}\right|^2 \left|(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = k^4 a^6 \left|\frac{K-1}{K+2}\right|^2 \left|(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right|^2}$$

(polarization)

Specific geometry:

$$\hat{k}_0 = \hat{z}$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\vec{\varepsilon}_0 = \hat{x}\cos\phi_0 + \hat{y}\sin\phi_0$$

$$\hat{r}\times\vec{\varepsilon}_0 = -\cos\theta\sin\phi_0\,\hat{x} + \cos\theta\cos\phi_0\,\hat{y} + \sin\theta(\cos\phi\cos\phi_0 + \sin\phi\sin\phi_0)\,\hat{z}$$

$$\boxed{\hat{r}\times\vec{\varepsilon}_0 = \cos\theta\,\hat{\phi}_0 + \sin\theta\,\cancel{\cos}(\phi-\phi_0)\,\hat{z}}$$

$-\sin\theta\sin(\phi-\phi_0)$.

Unpolarized incident → average over ϕ_0

$$|\hat{r}\times\vec{\varepsilon}_0|^2 = \cos^2\theta + \sin^2\theta\,\langle\cos^2(\phi-\phi_0)\rangle^{\frac{1}{2}}$$

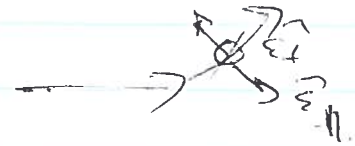
$$\longrightarrow \cos^2\theta + \tfrac{1}{2}\sin^2\theta = \tfrac{1}{2}(1+\cos^2\theta)$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left|\frac{k-1}{k+2}\right|^2 k^4 a^6\cdot\frac{1}{2}(1+\cos^2\theta)}$$

$$\boxed{\sigma = \frac{8\pi}{3}\left|\frac{k-1}{k+2}\right|^2 k^4 a^6}$$

Polarized detection let $\phi = 0$ (azimuthal symmetry).

$\hat{k}_0 = \hat{z} \qquad \hat{r} = \hat{x}\sin\theta + \hat{y}\cos\theta$



$\vec{\varepsilon}_\perp = \hat{y} \qquad \boxed{\vec{\varepsilon}_\parallel = \hat{z}\sin\theta - \hat{x}\cos\theta}$

$\vec{E}_{sc} \propto (\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}$

$\vec{\varepsilon}^{\,*}\cdot\left((\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right) = \vec{\varepsilon}^{\,*}\cdot\left[\vec{\varepsilon}_0(\hat{r}\cdot\hat{r}) - \hat{r}(\hat{r}\cdot\vec{\varepsilon}_0)\right]$

$= \vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0 - (\hat{r}\cdot\vec{\varepsilon}_0)\cancel{(\hat{r}\cdot\vec{\varepsilon}^{\,*})} = \underline{\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0}$

$\left(\frac{d\sigma_\perp}{d\Omega}\right) = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \left|\hat{y}\cdot(\cancel{\hat{x}\cos\phi_0} + \hat{y}\sin\phi_0)\right|^2$

$$\boxed{\frac{d\sigma_\perp}{d\Omega} = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \sin^2\phi_0}$$

$\left(\frac{d\sigma_\parallel}{d\Omega}\right) = \ldots \left|(\cancel{\hat{z}\sin\theta} - \hat{x}\cos\theta)\cdot(\hat{x}\cos\phi_0 + \cancel{\hat{y}\sin\phi_0})\right|^2$

$$\boxed{\frac{d\sigma_\parallel}{d\Omega} = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \cos^2\theta\cdot\cos^2\phi_0}$$

unpolarized. $\langle\cos^2\phi_0\rangle = \langle\sin^2\phi_0\rangle = 1/2$

$\sigma \propto \omega^4 \propto \frac{1}{\lambda^4}$

$\frac{\text{blue}}{\text{red}} = \frac{(700)^4}{(400)^4} = \frac{2401}{256} = 9.37 = \frac{1}{0.1066}$

$$\vec{E}_{in} = \frac{3}{K_e+2}\vec{E}_0 \qquad \vec{B}_{in} = \frac{3K_m}{K_m+2}\vec{B}_0$$

Conductivity

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = 3\varepsilon_0 \left(\frac{K_e-1}{K_e+2}\right)\vec{E}_0 \qquad K_e \to \infty$$

$$\vec{M} = \chi_m \vec{H} = \frac{3}{\mu_0}\left(\frac{K_m-1}{K_m+2}\right)\vec{B}_0 \qquad K_m \to 0$$

$$\boxed{\frac{1}{4\pi\varepsilon_0}\vec{p} = \vec{E}_0 a^3} \qquad \boxed{\frac{\mu_0}{4\pi}\vec{m} = -\frac{1}{2}\vec{B}_0 a^3}$$

$$\frac{m}{p} = \frac{-\frac{1}{2}\frac{4\pi}{\mu_0}B_0 a^3}{4\pi\varepsilon_0 E_0 a^3} = -\frac{1}{2}c^2\frac{B_0}{E_0} = -\frac{1}{2}c$$

$$\boxed{\frac{m/c}{p} = -\frac{1}{2}}$$

Both important

(even $ka \ll 1$).

$$\vec{E}_{sc} = \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left[(\hat{r}\times\vec{p})\times\hat{r} - \hat{r}\times\left(\frac{\vec{m}}{c}\right)\right]$$

(9.19) (9.36)

$$\vec{\varepsilon}^{\,*}\cdot\vec{E}_{sc} = \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\,\vec{\varepsilon}^{\,*}\cdot\left((\hat{r}\times\vec{p})\times\hat{r} - \hat{r}\times\frac{\vec{m}}{c}\right)$$

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$$\vec{\varepsilon}^{\,*}\cdot\left[(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right] = \vec{\varepsilon}^{\,*}\cdot\left[\vec{\varepsilon}_0(\hat{r}\cdot\hat{r}) - \cancel{\hat{r}}(\hat{r}\cdot\vec{\varepsilon}_0)\right]$$

$$= \vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0$$

$$\underline{\vec{B}_0 \sim \hat{k}_0\times\vec{\varepsilon}_0}$$

$$\vec{\varepsilon}^{\,*}\cdot\left[\hat{r}\times(\hat{k}_0\times\vec{\varepsilon}_0)\right] = (\vec{\varepsilon}^{\,*}\times\hat{r})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)$$

$$= -(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)$$

$$\boxed{\frac{d\sigma}{d\Omega} = k^4 a^6 \left| \underbrace{(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0)}_{\text{electric}} - \underbrace{\tfrac{1}{2}(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)}_{\text{magnetic}} \right|^2} \quad (10.14)$$

$\underline{(-1)^3}$ $\quad$ $\underline{\vec{m} = -\tfrac{1}{2}\vec{B}_0 a^3}$ $\quad$ $\underline{\vec{E} = -\hat{r}\times\vec{m}}$ $\quad$ $\cancel{\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0}$

$\underline{-(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)}$