2/26/2016.

$\vec{E}_{inc}$

$\vec{B}_{inc} = 0$

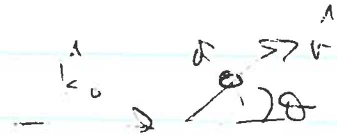
$$\vec{p} = 4\pi\epsilon_0 \vec{E}_0 a^3 \qquad \vec{m} = -\frac{1}{2}\frac{4\pi}{\mu_0}\vec{B}a^3$$

$$\vec{E}_{sc} = \frac{k^2}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\left[(\hat{r}\times\vec{p})\times\hat{r} - \hat{r}\times\frac{\vec{m}}{c}\right]$$

$$\vec{\epsilon}^{\,*}\cdot\vec{E}_{sc} = \ldots\left[\vec{\epsilon}^{\,*}\cdot\vec{\epsilon}_0 - \frac{1}{2}(\hat{r}\times\vec{\epsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\epsilon}_0)\right]$$

$$\frac{d\sigma}{d\Omega} = k^4 a^6\left|(\vec{\epsilon}^{\,*}\cdot\vec{\epsilon}_0) - \frac{1}{2}(\hat{r}\times\vec{\epsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\epsilon}_0)\right|^2$$

lots of geometry



$$\boxed{\hat{k}_0 = \hat{z} \qquad \hat{r} = \hat{z}\cos\theta + \hat{x}\sin\theta}$$

$$\vec{\varepsilon}_\perp = \hat{y} \qquad \vec{\varepsilon}_\parallel = \hat{z}\sin\theta - \hat{x}\cos\theta$$

$$\hat{\varepsilon}_0 = \hat{x}\cos\phi_0 + \hat{y}\sin\phi_0$$

(∥) $\vec{\varepsilon}_\parallel \cdot \hat{\varepsilon}_0 = -\cos\theta \cdot \cos(\phi - \phi_0)$

$$\underbrace{(\hat{r}\times\vec{\varepsilon}_\parallel)}_{\hat{y}} \cdot (\hat{k}_0 \times \vec{\varepsilon}_0) = \cancel{(\hat{z}\cos\theta + \hat{x}\sin\theta)\times(}$$

$$= -\cos(\phi - \phi_0)$$

(⊥) $\vec{\varepsilon}_\perp \cdot \vec{\varepsilon}_0 = -\sin(\phi - \phi_0)$

$$(\hat{r}\times\vec{\varepsilon}_\perp)\cdot(\hat{k}_0\times\vec{\varepsilon}_0) = -\cos\theta \cdot \sin(\phi - \phi_0)$$

$$\frac{d\sigma_\parallel}{d\Omega} = k^4 a^6 \left(\cos\theta - \frac{1}{2}\right)^2 \cos^2(\phi - \phi_0)$$

$$\frac{d\sigma_\perp}{d\Omega} = k^4 a^6 \left(1 - \frac{1}{2}\cos\theta\right)^2 \sin^2(\phi - \phi_0)$$

Unpolarized, $\langle \cos^2(\alpha-\alpha_0)\rangle = \langle \sin^2(\alpha-\alpha_0)\rangle = 1/2$

$(\parallel)$ $\quad \frac{1}{2}(k^4a^6)\left(\cos\theta - \frac{1}{2}\right)^2$

$(\perp)$ $\quad \frac{1}{2}k^4a^6\left(1-\frac{1}{2}\cos\theta\right)^2$

Total $\quad \boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2}(ka)^4a^2\left[\frac{5}{4}(1+\cos^2\theta) - 2\cos\theta\right]}$

1 from ele.. $\quad$ interference
$\frac{1}{4}$ from mag..

$\sigma = \frac{5}{4}\cdot k^4a^6\cdot\frac{8\pi}{3}$ $\quad \boxed{\sigma = \frac{10\pi}{3}k^4a^6}$

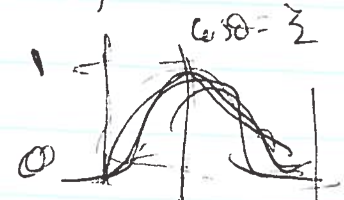
Asymmetry:

Forwards $\quad \int_0^1 d\mu\left[\frac{5}{8}(1+\mu^2) - \mu\right] = \frac{5}{8}\left(1+\frac{1}{3}\right) - \frac{1}{2} = \frac{1}{3}$

Backwards $\quad \int_{-1}^0 d\mu\left[\frac{5}{2}(1+\mu^2) - \mu\right] = \frac{5}{8}\left(1+\frac{1}{3}\right) + \frac{1}{2} = \frac{4}{3}$

$$\Pi = \frac{(\perp)-(\parallel)}{(\perp)+(\parallel)} = \frac{\left(1-\frac{1}{2}\cos\theta\right)^2 - \left(\cos\theta-\frac{1}{2}\right)^2}{\left(1-\frac{1}{2}\cos\theta\right)^2 + \left(\cos\theta-\frac{1}{2}\right)^2}$$

$$= \frac{3\sin^2\theta}{5(1+\cos^2\theta) - 8\cos\theta}$$



Multipole scattering

circularly polarized incident —

$$\vec{E}_0 = \vec{\varepsilon}_\pm E_0 e^{i\vec{k}\cdot\vec{x}} = (\hat{x} \pm i\hat{y}) e^{ikz} \qquad \Big|\; |E_0|^2 = 2$$

$$\vec{\nabla}\times\vec{E} = i\vec{k}_0\times\vec{\varepsilon}_\pm E_0 e^{i\vec{k}\cdot\vec{x}}$$

$$= ik\hat{z}\times(\hat{x}\pm i\hat{y})e^{ikz} = \pm k(\hat{x}\pm i\hat{y})e^{ikz}$$

$$= -\frac{\partial\vec{B}}{\partial t} = +i\omega\vec{B} = ick\vec{B} \qquad \boxed{c\vec{B}_0 = \mp i\vec{E}_0}$$

Write

$$\vec{E}_0 = \sum_{l,m}\left[a^\pm_{lm} j_l(kr)\vec{X}_{lm} + \frac{i}{k} b^\pm_{lm}\vec{\nabla}\times(j_l\vec{X}_{lm})\right]$$

$$c\vec{B}_0 = \sum_{l,m}\left[-\frac{i}{k}a^\pm_{lm}\vec{\nabla}\times(j_l\vec{X}_{lm}) + b^\pm_{lm} j_l\vec{X}_{lm}\right]$$

$$\int d\Omega\, \vec{X}^*_{lm}\cdot\vec{E}_0 = a^\pm_{lm} j_l(kr)$$

$$\int d\Omega\, \vec{X}_{lm}\cdot(c\vec{B}_0) = b^\pm_{lm} j_l(kr)$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - \frac{i}{r}\hat{r}\times\vec{L} \qquad \vec{\nabla}\times(j\vec{X}) = \left(\frac{dj}{dr}\right)(\hat{r}\times\vec{X}) - \frac{ij}{r}(\hat{r}\times\vec{L})\times\vec{X}$$

$$\rightarrow (\hat{r}\times\vec{L})\times\vec{L}\,Y \quad \rightarrow(\hat{r}L^2 + \hat{r}\times\vec{L})\,Y$$

$$\hat{r} \qquad \hat{r}\times\vec{\nabla}$$

$$\int d\Omega\, \vec{X}^{*}_{lm} \cdot \vec{E} = \int d\Omega \left(\frac{\vec{L} Y_{lm}}{\sqrt{l(l+1)}} \right)^{*} \cdot (\hat{x} \pm i\hat{y})\, e^{ikz}$$

$$= \int d\Omega \left[\frac{(\hat{x} \mp i\hat{y}) \cdot \vec{L} Y_{lm}}{\sqrt{l(l+1)}} \right]^{*} e^{ikz}$$

$$e^{ikz} = \sum_{l'=0}^{\infty} i^{l'} \sqrt{4\pi(2l'+1)}\; j_{l'}(kr)\, Y_{l',0}$$

$$e^{i\vec{k}\cdot\vec{x}} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} 4\pi i^{l} j_l(kr)\, Y^{*}_{lm}(\hat{k})\, Y_{lm}(\hat{x}) \quad (10.43) \quad \hat{k} = \hat{z}$$

$$j_l(kr)\, a^{\pm}_{lm} = \sum_{l'=0}^{\infty} \int d\Omega \left(\frac{L_{\mp} Y_{lm}}{\sqrt{l(l+1)}} \right)^{*} \sqrt{4\pi(2l'+1)}\; i^{l'} j_{l'}(kr)\, Y_{l',0}(\theta,\phi)$$

$l' = l$.

$$a^{\pm}_{lm} = i^{l} \frac{\sqrt{4\pi(2l+1)}}{\sqrt{l(l+1)}} \int d\Omega\, (L_{\mp} Y_{lm})^{*}\, Y_{l,0}$$

$$L_{\mp} Y_{lm} = Y_{l0} \qquad (m = \pm 1)$$

$$L_{-} Y_{l1} = \sqrt{(l+m)(l-m+1)}\, Y_{l0} = \sqrt{(l+1)l}\, Y_{l0} = \sqrt{l(l+1)}\, Y_{l,0}$$

$$L_{+} Y_{l,-1} = \sqrt{(l-m)(l+m+1)}\, Y_{l0} = \sqrt{(l+1)l}\, Y_{l0} = \sqrt{l(l+1)}\, Y_{l0}.$$

$$a^{\pm}_{lm} = i^l \sqrt{4\pi(2l+1)}\ \delta_{m,\pm 1}$$

$$b^{\pm}_{lm} = \mp i\, a^{\pm}_{lm}$$

(10.55)

(incident)

$$\vec{E}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr)\vec{X}_{l,\pm 1} \pm \frac{1}{k} \vec{\nabla}\times\left(j_l \vec{X}_{l,\pm 1}\right)\right]$$

$$c\vec{B}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left(-\frac{i}{k} \vec{\nabla}\times\left(j_l \vec{X}_{l,\pm 1}\right) \pm i\, j_l \vec{X}_{l,\pm 1}\right)$$