3|7|2016

Recall

Conducting sphere. $\vec{p} = 4\pi\varepsilon_0 \vec{E}_0 a^3$

$(\vec{E}_{inside} = 0)$

$\vec{m} = -\frac{1}{2} \cdot \frac{4\pi}{\mu_0} \vec{B}_0 a^3$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}(ka)^4 \cdot a^2 \left[\frac{5}{4}(1+\cos^2\theta) - 2\cos\theta\right]$$

$$\sigma = \frac{10\pi}{3} k^4 a^6$$

Multipole expansion $\sum a_l f_l \vec{X}_{lm} \; \cdots$

circular polarized incident

$$\vec{E}_0 = E_0 \vec{\varepsilon}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} = (\hat{x} \pm i\hat{y}) e^{ikz} \qquad |E_0|^2 = 2$$

$$\vec{E}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr)\vec{X}_{l,\pm 1} \pm \frac{1}{k} \vec{\nabla}\times(j_l \vec{X}) \right]$$

$$c\vec{B}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[-\frac{i}{k} \vec{\nabla}\times(j_l \vec{X}) \mp i\, j_l(kr)\vec{X}_{l,\pm 1} \right]$$

(10.55)

$c\vec{B}_0 = \mp i \vec{E}_0$ $\qquad$ $c\vec{B}_0 = (\hat{z}) \times \vec{E}_0$

②

Outgoing $h_\ell^{(1)}(kr)$ $A_\ell = i^\ell \sqrt{4\pi(2\ell+1)} \cdot \frac{1}{2}\alpha_\ell$

spherically symmetric scatterer (doesn't couple different m's)

$$\vec{E}_{sc} = \sum_{\ell=1}^{\infty} i^\ell \sqrt{4\pi(2\ell+1)} \left[\frac{1}{2}\alpha_\ell^{\pm} h_\ell^{(1)}(kr)\, \vec{X}_{\ell,\pm 1} \pm \frac{1}{2}\beta_\ell^{\pm} \frac{i}{k} \vec{\nabla} \times \left(h_\ell \vec{X}_\ell \right) \right]$$

$$c\vec{B}_{sc} = \sum_{\ell=1}^{\infty} i^\ell \sqrt{4\pi(2\ell+1)} \left[-\frac{i}{k} \frac{1}{2}\alpha_\ell \vec{\nabla} \times \left(h_\ell \vec{X}_\ell \right) \mp i \frac{1}{2}\beta_\ell^{\pm} h_\ell^{(1)}(kr)\, \vec{X}_{\ell,\pm 1} \right]$$

(10.57)

$$\langle \vec{S} \rangle = \frac{1}{2} \vec{E} \times \vec{H}^*$$

$\vec{r}, t = 0$ $\quad |e^{-i\omega t}|^2 = 1.$

$r \to a$

$$P_{scatt} = \int d^2a\, \hat{r} \cdot \mathrm{Re}\left[\frac{1}{2} \vec{E} \times \vec{H}^* \right]$$

$$\vec{E} \cdot (\vec{H}^* \times \hat{r}) = -\vec{E}_{sc} \cdot (\hat{r} \times \vec{H}_{sc})$$

$$P_{scatt} = -\frac{a^2}{2\mu_0} \int d\Omega\, \mathrm{Re}\left[\vec{E}_{sc} \cdot (\hat{r} \times \vec{B}_{sc}^*) \right] \quad (10.58)$$

$$\boxed{P_{abs} = \frac{a^2}{2\mu_0} \int d\Omega\, \mathrm{Re}\left[\vec{E} \cdot (\hat{r} \times \vec{B}^*) \right]} \quad (10.59)$$

Need to know $\boxed{\vec{\nabla}\times\left(f(r)\vec{X}_{\ell m}\right)}$ HW. $\qquad \vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - \frac{i}{r}\hat{r}\times\vec{L}$

$$\vec{\nabla}\times\left(f(r)\vec{X}_{\ell m}\right) = \hat{r}\frac{\partial f}{\partial r}\times\vec{X} - \frac{if}{r}\left(\hat{r}\times\vec{L}\right)\times\vec{X}$$

$$\left(\hat{r}\times\vec{L}\right)\times\vec{L} = \hat{r}_j L_i L_j - \hat{r}_i L_j L_j$$

$$= \hat{r}_j\left(L_j L_i + i\varepsilon_{ijk}L_k\right) - \hat{r}_i L^2$$

$$= i\left(\hat{r}\times\vec{L}\right) - \hat{r}L^2$$

$$\left(\hat{r}\times\vec{L}\right)\times\vec{X} = i\hat{r}\times\vec{X} - \hat{r}\frac{L^2 Y_{\ell m}}{\sqrt{\ell(\ell+1)}} \qquad (10.60)$$

$$\vec{\nabla}\times\left(f\vec{X}_{\ell m}\right) = \underbrace{\left(\frac{\partial f}{\partial r} + \frac{f}{r}\right)}_{\frac{1}{r}\frac{\partial}{\partial r}(rf)}\hat{r}\times\vec{X} + i\frac{\sqrt{\ell(\ell+1)}}{r}\hat{r}f Y_{\ell m}$$

$$\vec{E}_{sc}\cdot\left(\hat{r}\times\vec{B}_{sc}^{*}\right) = \sum_{\ell,\ell'} i^{\ell}(-i)^{\ell'}\sqrt{4\pi(2\ell+1)}\sqrt{4\pi(2\ell'+1)}$$

$$\left(\frac{1}{2}\alpha_\ell h_\ell\vec{X}_\ell \pm \frac{1}{k}\frac{1}{2}\beta_\ell\vec{\nabla}\times\left(h\vec{X}\right)\right)$$

$$\cdot\left[\hat{r}\times\left(\frac{-i}{k}\frac{1}{2}\alpha_\ell'\vec{\nabla}\times\left(h_{\ell'}\vec{X}_{\ell'}\right) \mp i\frac{1}{2}\beta_\ell' h_\ell\vec{X}_{\ell'}\right)\right]^{*}$$

$$\boxed{\sigma_{scatt} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1)\left(|\alpha_l^{\pm}|^2 + |\beta_l^{\pm}|^2\right)}$$

10.61

$$\boxed{\sigma_{abs} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1)\left[2 - |\alpha_l+1|^2 - |\beta_l+1|^2\right]}$$

$$r \to \infty \quad \vec{E}_{sc} \to \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)}\,(-i)^{l+1}$$
$$\times \frac{e^{ikr}}{r}\left(\frac{1}{2}\alpha_l \vec{X}_{l,\pm1} + i\,\frac{1}{2}\beta_l \hat{r}\times\vec{X}_{l,\pm1}\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{r^2 \cdot \frac{1}{2Z_0}|\vec{E}_{sc}|^2}{\frac{1}{2Z_0}|\vec{E}_0|^2} = \frac{r^2|\vec{E}_{sc}|^2}{2}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{scatt} = \frac{\pi}{2k^2}\left|\sum_{l=1}^{\infty}\sqrt{2l+1}\left(\alpha_l \vec{X}_{l,\pm1} \pm i\,\beta_l\,\hat{r}\times\vec{X}_{l,\pm1}\right)\right|^2}$$

what it's good for

~~$\alpha = ?$~~ ~~$\beta = ??$~~

"Optical Theorem"