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$$\vec{E}_{sc} = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\frac{1}{2} \alpha_l h_l \vec{X}_l \pm \frac{i}{2} \beta_l \frac{1}{k} \vec{\nabla} \times \left(h_l \vec{X}_l \right) \right]$$

$$\sigma_{scatt} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1) \left(|\alpha_l|^2 + |\beta_l|^2 \right)$$

(10.61)

$$\sigma_{abs} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1) \left[2 - |1+\alpha_l|^2 - |1+\beta_l|^2 \right]$$

"Optical Theorem"

$$\sigma_{tot} = \sigma_{abs} + \sigma_{scatt}.$$

$$\sigma_{tot} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1) \Big[2 - (1+\alpha_l^*)(1+\alpha_l) - (1+\beta_l^*)(1+\beta_l) + |\alpha_l|^2 + |\beta_l|^2 \Big]$$

$$\cancel{2} - \cancel{1} - \alpha - \alpha^* - \cancel{|\alpha|^2} - \cancel{1} - \beta - \beta^* - \cancel{|\beta|^2} + \cancel{|\alpha|^2} + \cancel{|\beta|^2}$$

$$= -(\alpha + \alpha^*) - (\beta + \beta^*).$$

$$\boxed{\sigma = -\frac{\pi}{k^2} \sum_{l=1}^{\infty} (2l+1)\, \mathrm{Re}(\alpha_l + \beta_l)}$$

Linear in α, β: amplitude.

Forward scattering amplitude. $\vec{E}_{sc} = \vec{F} \frac{e^{ikr}}{r}$

$$\vec{X}_{l,\pm 1} = \frac{\vec{L}\, Y_{lm}}{\sqrt{l(l+1)}} = \frac{\frac{1}{2}(L_+ + L_-)\hat{x} + \frac{1}{2i}(L_+ - L_-)\hat{y} + L_z \hat{z}}{\sqrt{l(l+1)}}\, Y_{l,\pm 1}$$

$m=+1$ $\quad Y_{l,1} \propto \sin\theta \quad L_z \to m. \quad \sin\theta \to 0$

$L_+ Y_{l,1} \sim Y_{l,2} \sim \sin^2\theta \to 0$

$L_- Y_{l,1} \sim Y_{l,0} \sim P_l(1)$

3

$$\vec{X}_{l,1}(\theta=0) = \frac{\frac{1}{2}(\hat{x}+i\hat{y})L_- Y_{l,1}}{\sqrt{l(l+1)}}$$

$$= \frac{\frac{1}{2}(\hat{x}+i\hat{y})\cancel{\sqrt{(l+m)(l-m+1)}}}{\cancel{\sqrt{l(l+1)}}}\sqrt{\frac{2l+1}{4\pi}}\,P_l(1)$$

$$= \frac{1}{2}(\hat{x}+i\hat{y})\sqrt{\frac{2l+1}{4\pi}}$$

$$\vec{X}_{l,-1} \rightarrow \frac{\frac{1}{2}(\hat{x}-i\hat{y})L_+}{\quad} \longrightarrow \frac{1}{2}(\hat{x}-i\hat{y})\sqrt{\frac{2l+1}{4\pi}}$$

$$\vec{E}_{sc} \rightarrow \sum_{l=1}^{\infty} \cancel{i^l}\sqrt{4\pi(2l+1)}\left((-i)^{l+1}\frac{e^{ikr}}{kr}\right) = \frac{e^{ikr}}{r}\vec{F}_{sc}.$$

$$\times\left(\frac{1}{2}\alpha_l\vec{X}_{l,\pm 1} \pm i\frac{1}{2}\beta_l\,\hat{r}\times\vec{X}_{l,\pm 1}\right)$$

$$\vec{F}_{sc}(\theta=0) = \frac{-i}{2k}\sum_{l=1}^{\infty}(2l+1)\cdot\frac{1}{2}\left(\alpha_l(\hat{x}\pm i\hat{y}) \pm i\beta_l\,\hat{z}\times(\hat{x}\pm i\hat{y})\right)$$

$$\hat{z}\times(\hat{x}\pm i\hat{y}) = \mp i(\hat{x}\pm i\hat{y})$$

$$\vec{F}_{sc} = \frac{-i}{4k}(\hat{x}\pm i\hat{y})\sum_{l=1}^{\infty}(2l+1)(\alpha_l+\beta_l)$$

<u>Normalized</u>: $\vec{\varepsilon}_f^{\,*} \cdot \vec{f}_{sc} = \dfrac{\vec{\varepsilon}_f^{\,*} \cdot \vec{F}_{sc}}{E_0}$

$$\vec{\varepsilon}_f^{\,*} \cdot \vec{f}_{sc}(\theta=0) = \frac{-i}{4k} \sum_{l=1}^{\infty} (2l+1)(\alpha_l + \beta_l)$$

$$\sigma_{tot} = -\frac{\pi}{k^2} \sum_{l=1}^{\infty} (2l+1)\, \mathrm{Re}(\alpha_l + \beta_l)$$

$$\boxed{\sigma_{tot} = \frac{4\pi}{k} \,\mathrm{Im}\left[\vec{\varepsilon}_0^{\,*} \cdot \vec{f}(\theta=0)\right]} \quad (10.139)$$

(different section).

$$\vec{E}_0 = (\hat{x} \pm i\hat{y})\, e^{ikz}$$

$$= \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr)\, \vec{X}_{l,\pm1} \pm \frac{1}{k} \vec{\nabla} \times \left(j_l \vec{X}_{l,\pm1} \right) \right]$$

$$\vec{E}_{sc} = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[\frac{1}{2}\alpha_l h_l^{(1)} \vec{X}_{l,\pm1} \pm \frac{1}{2}\beta_l \frac{1}{k} \vec{\nabla} \times \left(h_l \vec{X} \right) \right]$$

α_l, β_l : Boundary conditions

"Conducting" $\longrightarrow$ $\vec{E}_{\parallel}\big|_S = 0$ $\quad \vec{H}_{\perp}\big|_S = 0.$

Total = $\frac{1}{2}\alpha_l h^{(1)} + j_l = \frac{1}{2}\alpha_l h^{(1)} + \frac{1}{2}\left(h^{(1)} + h^{(2)}\right) = 0$

$\longrightarrow \frac{1}{2}\left(e^{2i\delta_l} - 1\right) h^{(1)} + \frac{1}{2}\left(h^{(2)} + h^{(1)}\right)$

$\longrightarrow \frac{1}{2} h^{(2)} + \frac{1}{2} e^{2i\delta_l} h^{(1)}$

incoming $\qquad$ outgoing

(Almost) as easy. $\quad \vec{E}_{\parallel}\big|_S = Z_S\, \hat{r} \times \vec{H}_{\parallel}\big|_S$

Z_S = surface impedance.