3/16/2016 $S_1 = \{\text{circle of radius } a\}$ $\quad ka >> 1.$

$$\vec{k}_0 = k\left(\hat{x}\sin\theta_0\cos\phi_0 + \hat{y}\sin\theta_0\sin\phi_0 + \hat{z}\cos\theta_0\right)$$

$$\psi_{scatt} = \frac{-ik}{4\pi}\frac{e^{ikr}}{r}\int_0^a \rho' d\rho' d\phi' \underline{(\cos\theta_0 + \cos\theta)}\ \underline{e^{i\vec{k}_0\cdot\vec{x}'}\, e^{-i\vec{k}''\cdot\vec{x}'}}$$

$$\exp\left[ik\rho'\sin\theta_0(\cos\phi_0\cos\phi' + \sin\phi_0\sin\phi') - ik\rho'\sin\theta(\cos\phi\cos\phi' + \sin\phi\sin\phi')\right]$$

$$= (\xi_x)\cos\phi' + (\xi_y)\sin\phi'$$

$$= \vec{\xi}\cdot\hat{\rho}' = \xi\cos(\phi' - \delta)$$

$$\xi_x = \sin\theta_0\cos\phi_0 - \sin\theta\cos\phi$$

$$\xi_y = \sin\theta_0\sin\phi_0 - \sin\theta\sin\phi$$

$$\xi^2 = \xi_x^2 + \xi_y^2 = \sin^2\theta_0 + \sin^2\theta - 2\sin\theta_0\sin\theta\cos(\phi - \phi_0)$$

$$= |\vec{\Delta\theta}|^2 \qquad \theta \ll 1$$

$$\psi(x) = \frac{-ik}{4\pi}\frac{e^{ikr}}{r}\left(\cos\theta_0+\cos\theta\right)\int_0^a \rho'd\rho' \int d\phi' \, e^{ik\rho'\xi\cos(\phi'-\phi)}$$

$$= (\;\cdots\;)\int_0^a \rho' d\rho' \cdot 2\pi\, J_0(k\rho'\xi)$$

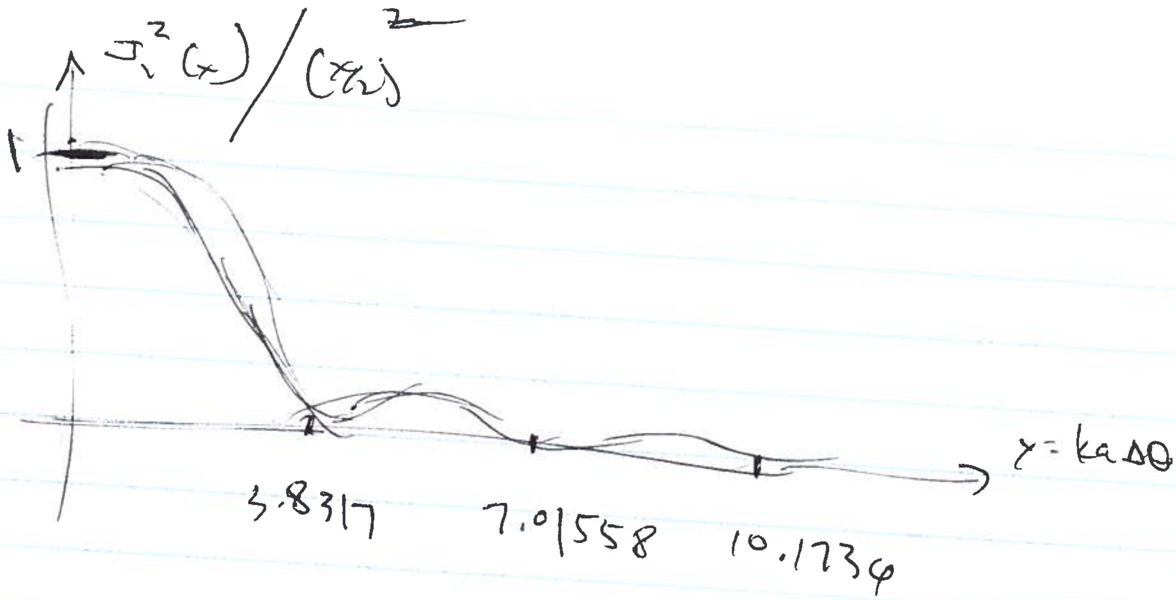
$$= (\;\cdots\;)\,\frac{2\pi a^2}{ka\xi}\, J_1(ka\xi)$$

$$\boxed{\psi(\vec{x}) = -ia^2\left(\frac{\cos\theta_0+\cos\theta}{2}\right)\frac{J_1(ka\xi)}{ka\xi}\frac{e^{ikr}}{r}}$$

$$|\psi|^2 = I_0\left|\frac{J_1(ka\xi)}{ka\xi/2}\right|^2$$

$$ka\xi = x_{11} = 3.832 = \frac{2\pi}{\lambda}a\cdot\Delta\theta$$

$$\text{diameter} = 2\,\Delta\theta = \frac{3.832}{\pi}\cdot\frac{\lambda}{a} = 1.21967$$



$\underline{x_{11} \to \theta}$ 0.162215

$\underline{x_{12} \to \theta}$ 0.09007

Babinet's principle

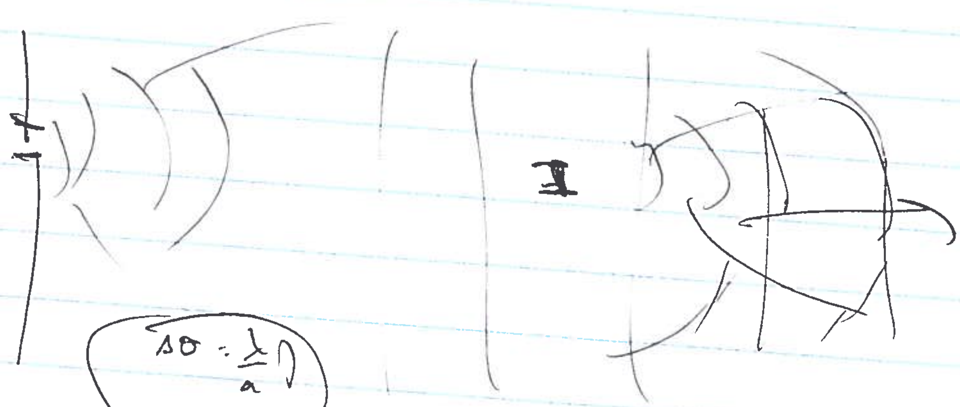
Apertures (A) = Screen (B)
Screen (A) = Apertures (B).

$$\psi(\vec{x}) = \frac{1}{4\pi}\int_{\text{Apertures}} d^2x' \left(\psi_0 \frac{\partial G}{\partial n'} + G \frac{\partial \psi_0}{\partial n'}\right)$$

$$\left.\begin{array}{l} \psi_A = \dfrac{1}{4\pi}\displaystyle\int_{\text{Apertures (A)}} d^2x' \\ \psi_B = \dfrac{1}{4\pi}\displaystyle\int_{\text{Screen (A)}} d^2x \end{array}\right\} \psi_A + \psi_B = \frac{1}{4\pi}\int_{\text{All}} d^2x' \left(\cdot\cdot \right) = \psi_0 .$$

$$\psi_B = \psi_0 - \psi_A$$

§ 10.8



$\Delta\theta = \frac{\lambda}{a}$