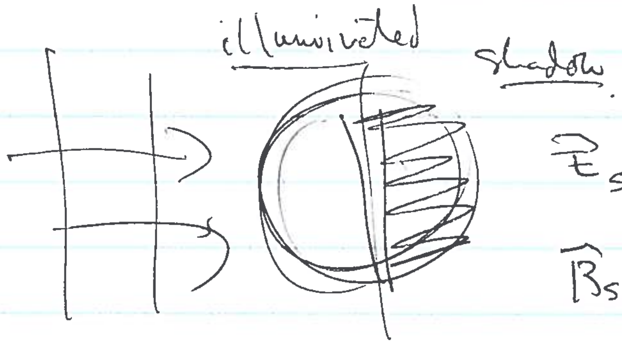
3|21|2010

$$\vec{\epsilon}^{*}\cdot\vec{F}_{scatt} = \frac{ik}{4\pi}\int_{S_1} d^2a'\,\vec{\epsilon}^{*}\cdot\left[(\hat{n}'\times c\vec{B}_{sc}) + \hat{r}\times(\hat{n}'\times\vec{E}_{sc})\right]e^{-ik\hat{r}\cdot\vec{x}'}$$



(10.93)

$$\vec{E}_{scatt} = -\vec{E}_0$$

$$\vec{B}_{scatt} = -\vec{B}_0$$

$$\vec{E}_0 = \hat{\epsilon}_0 E_0\, e^{i(\vec{k}_0\cdot\vec{x}-\omega t)}$$

$$c\vec{B}_0 = \hat{k}_0\times\vec{E}_0$$

$$\underline{\vec{\epsilon}^{*}\cdot\vec{F}_{shadow}} = -\frac{ikE_0}{4\pi}\int d^2a'\, e^{i\vec{k}_0\cdot\vec{x}'}\,e^{-i\vec{k}\cdot\vec{x}'}\,\vec{\epsilon}^{*}\cdot\left[\hat{n}'\times(\hat{k}_0\times\vec{\epsilon}_0) + \hat{k}\times(\hat{n}'\times\vec{\epsilon}_0)\right]$$

②

$$\hat{n}' \times (\hat{k}_0 \times \vec{\varepsilon}_0) + \hat{k} \times (\hat{n}' \times \vec{\varepsilon}_0)$$

$$= \underline{\hat{k}_0 (\hat{n}' \cdot \vec{\varepsilon}_0)} - \underline{\underline{\vec{\varepsilon}_0 (\hat{n}' \cdot \hat{k}_0)}}$$

$$+ \hat{n}' (\hat{k} \cdot \vec{\varepsilon}_0) - \underline{\underline{\vec{\varepsilon}_0 (\hat{k} \cdot \hat{n}')}}$$

insert

$$+ \hat{n}' (\hat{k}_0 \cdot \vec{\varepsilon}_0)$$

$$= (\hat{k}_0 + \hat{k}) \times (\hat{n}' \times \vec{\varepsilon}_0) + \hat{k}_0 (\hat{n}' \cdot \vec{\varepsilon}_0)$$

$$\vec{\varepsilon}^{\,*} \cdot \vec{F}_{\text{shadow}} = \frac{-ikE_0}{4\pi} \int_{\text{shadow}} d^2 a' \, \vec{\varepsilon}^{\,*} \cdot \Big[(\hat{k}_0 + \hat{k}) \times (\hat{n}' \times \vec{\varepsilon}_0) + \hat{k}_0 (\hat{n}' \cdot \vec{\varepsilon}_0) \Big] \times e^{i(\vec{k} - \vec{k}_0) \cdot \vec{x}'}$$

$\boxed{ka \gg 1}$ Away from forward direction, $\hat{k} \simeq \hat{k}_0$ $(\hat{r} \simeq \hat{z})$

phase oscillates rapidly. → destructive interference

$$\boxed{\Delta\theta \lesssim 1/ka}$$

$$(\vec{k} - \vec{k}_0) \cdot \vec{x}' = ka \Big[\underbrace{(\cos\theta - 1)}_{-\frac{1}{2}\theta^2 \sim \left(\frac{1}{ka}\right)^2} \cos\theta' + \underbrace{\sin\theta}_{\sim \frac{1}{ka}} \sin\theta' \cos(\phi - \phi') \Big]$$

$$\vec{\varepsilon}^{\,*} \cdot \hat{k}_0 \sim \sin\theta \sim \frac{1}{ka} \ll 1.$$

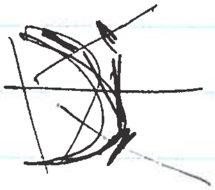
$$(\hat{k} + \hat{k}_0) \times (\hat{n}' \times \vec{\varepsilon}_0) \simeq 2\hat{k}_0 \times (\hat{n}' \times \vec{\varepsilon}_0)$$

$$= 2\hat{n}' \cancel{(\hat{k}_0 \cdot \vec{\varepsilon}_0)} - \underline{2\vec{\varepsilon}_0 (\hat{k}_0 \cdot \hat{n}')}$$

$$\vec{\varepsilon}^{\,*}\cdot\vec{F}_{\text{shadow}} \simeq \frac{ikE_0}{2\pi}\,(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0)\int_{\text{shadow}} d^2a'\,(\hat{k}_0\cdot\hat{n}')\,e^{i(\vec{k}-\vec{k}_0)\cdot\vec{x}'}$$

$$(\vec{k}_0-\vec{k})\cdot\vec{x}' = kz'\cancel{(1-\cos\theta)} - kx'\sin\theta\cos\phi - ky'\cdot\sin\theta\sin\phi$$

$$= \underline{-\vec{k}_\perp\cdot\vec{x}'_\perp}$$



$$(\hat{k}_0\cdot\hat{n}')\,d^2a' = d^2a'\cdot\cos\theta' = d^2a'_\perp$$

$$\boxed{\vec{\varepsilon}^{\,*}\cdot\vec{F}_{\text{shadow}} = \frac{ikE_0}{2\pi}\,(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0)\int_{A_\perp} d^2a'\,e^{-i\vec{k}_\perp\cdot\vec{x}'_\perp}}$$

Universal

(10.125)

sphere.
$$\int_0^a \rho'd\rho'\int d\phi'\;e^{-ik\rho'\sin\theta\cos(\phi-\phi')}$$

$$= \int_0^a \rho'd\rho'\cdot 2\pi J_0(k\rho'\sin\theta)$$

$$= \underline{2\pi a^2}\cdot\frac{J_1(ka\sin\theta)}{ka\sin\theta}$$

$$\boxed{\vec{\varepsilon}^{\,*}\cdot\vec{F}_{\text{shadow}} = ika^2\cdot E_0\,(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0)\left(\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right)}$$

(10.127)

Illuminated : conductor $\rightarrow \vec{E}_{\parallel}\big|_S = 0 \quad \vec{B}_{\perp}\big|_S = 0.$

$$\vec{E}_{\parallel,\text{scatt}} = -\vec{E}_{\parallel,0} \qquad \vec{B}_{\parallel,\text{scatt}} = +\vec{B}_{\parallel,0}$$

Specular reflection

Stationary phase approximation $\rightarrow$

Hard sphere classical.

$$\varepsilon^{*}\cdot\vec{F}_{\text{ill.}} = E_0 \; \boxed{\frac{1}{2}a} \; (\vec{\varepsilon}^{*}\cdot\vec{\varepsilon}_r)\, e^{-2ika\sin\theta/2}$$

$$\vec{\varepsilon}^{*}\cdot\vec{F}_{\text{shadow}} \sim \boxed{ka^2}$$ larger by $ka \gg 1.$

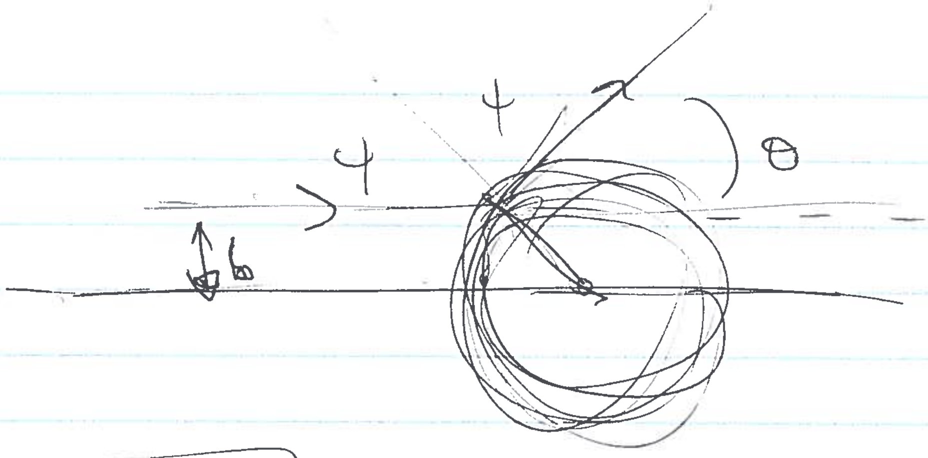
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{ill.}} = \frac{|\vec{\varepsilon}^{*}\cdot\vec{F}|^2}{|E_0|^2} = \frac{1}{4}a^2\,|\vec{\varepsilon}^{*}\cdot\vec{\varepsilon}_r|^2 \qquad \boxed{\pi a^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{shadow}} = k^2a^4\,|\vec{\varepsilon}^{*}\cdot\vec{\varepsilon}_0|^2 \left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]^2 \qquad \boxed{\pi a^2}$$

magic

$\boxed{\sin\theta\, d\theta\, d\phi} \rightarrow \sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right) \rightarrow \boxed{\cos\frac{\theta}{2}}$ geometry $\rightarrow \boxed{\sin\frac{\theta}{2}}$

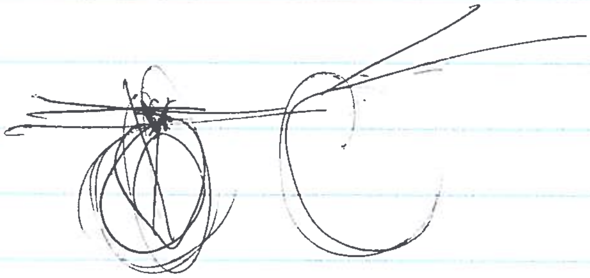
$\boxed{\frac{\text{Gaussian}}{\Delta\theta,\, \delta\phi}} \rightarrow \frac{1}{\sqrt{\sin\frac{\theta}{2}}}\;\frac{1}{\sqrt{\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}}}$



$$\boxed{b = a \sin\psi}$$

$$\theta + 2\psi = \pi \longrightarrow \boxed{\psi = \frac{\pi}{2} - \frac{\theta}{2}} \quad \sin\psi = \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$= \sin\frac{\pi}{2}\cdot\cos\frac{\theta}{2} \cancel{- \cos\frac{\pi}{2}\sin\frac{\theta}{2}}$$



$$dN = I_0 \cdot b\, db\, d\phi = \left(\frac{dN}{d\Omega}\right) \sin\theta\, d\theta\, d\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{dN/d\Omega}{I_0} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$= \frac{a\cdot\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}{\sin\theta}\cdot\left|\frac{d}{d\theta}\left(a\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right)\right| \qquad \boxed{= \frac{a^2}{4}}$$

$$= \frac{a^2}{\sin\theta}\cdot\cos\frac{\theta}{2}\cdot\frac{1}{2}\sin\frac{\theta}{2} = \frac{a^2}{4}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin\theta}\right)$$

Relativity   Chapter 11.

we have seen: maxwell → $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = (-\rho/\varepsilon_0)$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = (-\mu_0\vec{J})$$

wave @ speed $v=c$, w/r to??, as obs. by ??

Galilean relativity, $\vec{x} \to \vec{x} + \vec{v}t$

$$m_i\frac{d^2\vec{x}_i}{dt^2} = -\frac{\partial}{\partial x_i}\sum_{j\neq i} V(|\vec{x}_i - \vec{x}_j|).$$ unchanged.

$\Phi(\vec{x}+\vec{v}t, t)$ $\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$

$$\nabla^2\Phi - \frac{1}{c^2}\left(\xcancel{\frac{\partial^2\Phi}{\partial t^2} + 2\vec{v}\cdot\vec{\nabla}}\frac{\partial\Phi}{\partial t} + (\vec{v}\cdot\vec{\nabla})^2\Phi\right) = 0$$

Need: restrict to ~~special~~ observer

Different transformation

simultaneous on ground

⇒ not on train

time depends on observer