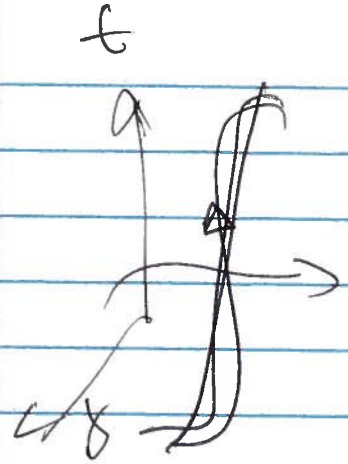
3/23/2016



$x^\mu(\lambda) = \begin{cases} t(\lambda) \\ x(\lambda) \end{cases}$ path in spacetime

$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = x$

$\boxed{\mu = 0, 1, 2, 3}$ $\boxed{\vec{x} = x^i \quad i = 1, 2, 3}$

"An <u>event</u> is such a little piece of time-and-space you can mail it through the slotted eye of a cat"
(Mystic Communion of Clocks)

$f(x^\mu) = f(x^\mu(\lambda))$: scalar function of scalar λ

$\frac{df}{d\lambda} = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{d\lambda} = (\partial_\mu f)\left(\frac{dx^\mu}{d\lambda}\right) \leftarrow$ vector

$\uparrow$ covector

transform differently

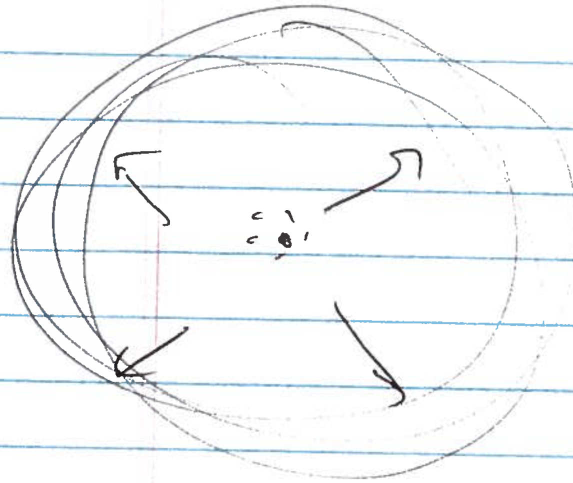
$$x'^{\mu} = x^{\mu}(x'^{\alpha})$$

$$\frac{dx^{\mu}}{d\lambda} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{dx'^{\alpha}}{d\lambda} \qquad \frac{\partial f}{\partial x^{\mu}} = \frac{\partial f}{\partial x'^{\beta}} \frac{\partial x'^{\beta}}{\partial x^{\mu}}.$$

$$\left[\frac{dx'^{\alpha}}{d\lambda} \cdot \frac{\partial x^{\mu}}{\partial x'^{\alpha}}\right]\left[\frac{\partial x'^{\beta}}{\partial x^{\mu}} \frac{\partial f}{\partial x'^{\beta}}\right]$$

$$= \frac{dx'^{\alpha}}{d\lambda} \cdot \underbrace{\left(\frac{\partial x'^{\beta}}{\partial x'^{\alpha}}\right)}_{\delta^{\beta}{}_{\alpha}} \frac{\partial f}{\partial x'^{\beta}} = \frac{\partial f}{\partial x'^{\beta}} \frac{dx'^{\beta}}{d\lambda}$$

Vector, Covector inverse transformation



$$|\vec{x}|^2 = c^2t^2$$
$$|\vec{x}'|^2 = c^2t'^2$$

flash expands at c for two different observers

$$\underbrace{c^2t^2 - |\vec{x}|^2 = c^2t'^2 - |\vec{x}'|^2}_{\text{invariant}} = 0.$$

let. $x \cdot y = \eta_{\mu\nu} x^\mu y^\nu$ $\qquad \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$\eta = g =$ "metric". (special relativity) signature

want to find. linear transformation Λ: $x' = \Lambda x$

$x'^\alpha = \Lambda^\alpha{}_\mu x^\mu$ such that

$$x' \cdot y' = \eta_{\alpha\beta} x'^\alpha y'^\beta = \eta_{\alpha\beta}(\Lambda^\alpha{}_\mu x^\mu)(\Lambda^\beta{}_\nu y^\nu)$$
$$= (\eta_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu) x^\mu y^\nu = \eta_{\mu\nu} x^\mu y^\nu$$

$$\Rightarrow \boxed{\eta_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = \eta_{\mu\nu}}$$

④

$$(\Lambda^T)_\mu{}^\alpha \eta_{\alpha\beta} \Lambda^\beta{}_\nu = \eta_{\mu\nu} \qquad \Lambda^T \eta \Lambda = \eta$$

<u>A near identity</u> $\quad \Lambda^\alpha{}_\mu = \delta^\alpha{}_\mu + \varepsilon^\alpha{}_\mu$

$$\eta_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = \eta_{\alpha\beta} (\delta^\alpha{}_\mu + \varepsilon^\alpha{}_\mu)(\delta^\beta{}_\nu + \varepsilon^\beta{}_\nu)$$

$$= \eta_{\alpha\beta} \delta^\alpha{}_\mu \delta^\beta{}_\nu + \eta_{\alpha\beta} \delta^\alpha{}_\mu \varepsilon^\beta{}_\nu$$
$$+ \eta_{\alpha\beta} \varepsilon^\alpha{}_\mu \delta^\beta{}_\nu + O(\varepsilon^2)$$

$$= \eta_{\mu\nu} + \eta_{\mu\beta} \varepsilon^\beta{}_\nu + \eta_{\alpha\nu} \varepsilon^\alpha{}_\mu$$

$$= \eta_{\mu\nu} + \varepsilon_{\mu\nu} + \varepsilon_{\nu\mu} = \eta_{\mu\nu}$$

$$\varepsilon_{\mu\nu} + \varepsilon_{\nu\mu} = 0$$

<u>Antisymmetric</u> $\quad$ <u>$4\times4 \rightarrow 6$ d.o.f.</u>