3/28/2016.

$$x' = \gamma(x + vt)$$
$$t' = \gamma\left(t + \frac{vx}{c^2}\right)$$

$v \ll c \longrightarrow$ $x' = x + vt$, $t' = t$

$$\Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta} = g_{\mu\nu}$$

$$(\Lambda^T) \eta \Lambda = \eta$$

$$\eta^{-1} \Lambda^T \eta \Lambda = 1 \qquad \eta^{-1} \Lambda^T \eta = \Lambda^{-1}$$

$$(\Lambda^{-1})^{\alpha}{}_{\beta} = (\eta^{-1})^{\alpha\mu} (\Lambda^T)_{\mu}{}^{\nu} (\eta)_{\nu\beta}$$
$$= \eta^{\alpha\mu} \Lambda^{T}{}_{\mu}{}^{\nu} \eta_{\nu\beta} = \underline{\Lambda^{T\alpha}{}_{\beta}}$$

$$\Lambda^{-1} = \eta^{-1} \Lambda^T \cdot \eta$$

rotation $\quad (\eta^2 = 1) \quad \underline{R^T = R^{-1}} \quad \theta \to -\theta$

Boost $\quad \Lambda^T = \Lambda \quad \eta^{-1}\Lambda\eta \Rightarrow \left(\begin{array}{c|c} + & - \\ \hline - & + \end{array}\right) \quad v \to -v,\ \zeta \to -\zeta.$

$(-)^2$

$$\Lambda = \exp(\vec{\theta} \cdot \vec{J} + \vec{\zeta} \cdot \vec{K})$$

②

<u>Tensors</u>. $x'^{\alpha} = \Lambda^{\alpha}{}_{\mu} x^{\mu}$ vector

$$T'^{\alpha\beta}{}_{\gamma\delta} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} \Lambda_{\gamma}{}^{\rho} \Lambda_{\delta}{}^{\sigma} \; T^{\mu\nu}{}_{\rho\sigma}.$$

since η can raise/lower any index, $T^{\alpha\beta\gamma\delta}$.

<u>Special Tensors</u> $\eta'_{\alpha\beta} = \Lambda_{\alpha}{}^{\mu} \Lambda_{\beta}{}^{\nu} \eta_{\mu\nu} = \eta_{\alpha\beta}$

$\varepsilon^{0123} = +1$ $\quad \varepsilon'^{\alpha\beta\gamma\delta} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} \Lambda^{\gamma}{}_{\rho} \Lambda^{\delta}{}_{\sigma} \; \varepsilon^{\mu\nu\rho\sigma}$

$= (\det\Lambda)\,(\varepsilon^{\alpha\beta\gamma\delta}) = \varepsilon^{\alpha\beta\gamma\delta}$

proper

$T^{\mu}{}_{\nu} = T^{\mu\alpha}{}_{\alpha\nu}$ $\quad T' = \Lambda\Lambda\Lambda T$

$\mathrm{Tr}\,T' = \mathrm{Tr}(\Lambda\Lambda\Lambda T) = \Lambda\Lambda(\mathrm{Tr}\,T)$

~~before~~ or ~~after~~

$T_{(\alpha\beta)} = \frac{1}{2}(T_{\alpha\beta} + T_{\beta\alpha})$ $\quad T_{(\alpha\beta\gamma)}$ $\quad \frac{(n+s)!}{(n)!\,s!}$ $\quad \cdot|\cdot\cdot|\cdot\cdot\cdot|$

$T_{[\alpha\beta]} = \frac{1}{2}(T_{\alpha\beta} - T_{\beta\alpha})$ $\quad T_{[\alpha\beta\gamma]}$ $\quad \frac{n(n-1)(n-2)}{3!}$

$\frac{n!}{a!\,(n-a)!}$

$x^\mu = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix}$,

$\vec{v} = \frac{d\vec{x}}{dt}$ is not a vector. (will see how it transf)

$d\tau^2 = dt^2 - \frac{1}{c^2}(d\vec{x})^2 = \frac{1}{c^2}\, dx \cdot dx$ scalar

along a path where $d\vec{x} = \vec{v}\,dt$ $\quad d\tau^2 = dt^2 - \left(\frac{\vec{v}\,dt}{c}\right)^2$

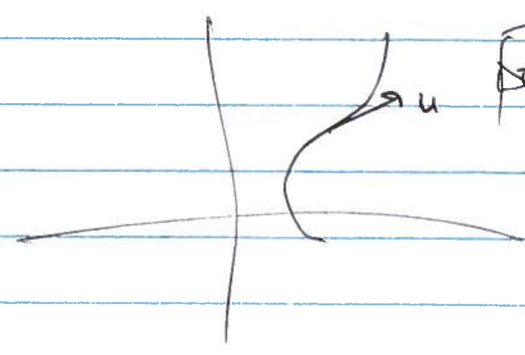
$$\boxed{d\tau = \frac{dt}{\gamma}} \qquad = \frac{dt^2}{\gamma^2}$$

$$u^\mu \equiv \frac{dx^\mu}{d\tau} = \begin{pmatrix} c\,\frac{dt}{d\tau} \\ \frac{d\vec{x}}{dt} \end{pmatrix} = \begin{pmatrix} c \cdot \gamma \cdot \frac{dt}{dt} \\ \gamma \cdot \frac{d\vec{v}}{dt} \end{pmatrix} = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix}$$

(4-vector velocity) $\quad$ $\boxed{\text{4-velocity}}$

$$u \cdot u = (\gamma c)^2 - (\gamma \vec{v})^2 = c^2 \gamma^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{c^2(1 - v^2/c^2)}{1 - v^2/c^2}$$

$$\boxed{u \cdot u = c^2}$$



$\boxed{\tau = \int d\tau}$

$$a^\mu = \frac{du^\mu}{d\tau}$$

$$u \cdot u = c^2$$

$$\frac{d}{d\tau}(u \cdot u) = 2u \cdot a = 0 \qquad \boxed{u \cdot a = 0}$$

at rest (proper). $\vec{v} = 0$ $\quad u = \begin{pmatrix} c \\ \vec{0} \end{pmatrix}$

$$a = \begin{pmatrix} 0 \\ \vec{a} \end{pmatrix}$$

$$a \cdot a = -|\vec{a}|^2_{\text{proper}}$$

---

constant proper acceleration $|\vec{a}|_{\text{proper}} = g.$

$$u \cdot u = (u^t)^2 - (u^x)^2 = c^2$$

$$a \cdot a = (a^t)^2 - (a^x)^2 = -g^2$$

$$u \cdot a = (u^t)(a^t) - (u^x)(a^x) = 0$$

remove x-components $\quad (a^x)^2 = (a^t)^2 + g^2 = a^2 + g^2$

$$(u^x)^2 = (u^t)^2 - c^2 = u^2 - c^2$$

$$u^2 a^2 = (u^t)^2 (a^t)^2 = (u^x)^2 (a^x)^2 = (u^2 - c^2)(a^2 + g^2)$$

$$\cancel{u^2 a^2} = \cancel{u^2 a^2} + u^2 g^2 - a^2 c^2 - g^2 c^2$$

$$\boxed{\frac{u^2}{c^2} - \frac{a^2}{g^2} = 1}$$

5

$$\boxed{\left(\frac{du}{d\tau}\right)^2 = g^2\left(\frac{u^2}{c^2}-1\right)}$$

$$\int_c^u \frac{du'}{\sqrt{\frac{u'^2}{c^2}-1}} = c\cdot\cosh^{-1}\left(\frac{u'}{c}\right)\Big|_c^u = c\cdot\cosh^{-1}\frac{u}{c} = g\tau$$

$$\boxed{u = c\cdot\cosh\left(\frac{g\tau}{c}\right)} = c\,\frac{dt}{d\tau}$$

$$\boxed{ct = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right)} \qquad ct \simeq \frac{c^{\cancel{2}}}{\cancel{g}}\,\frac{\cancel{g}\tau}{\cancel{c}} = c\tau$$

$$\left(\frac{dx}{d\tau}\right)^2 = \cancel{c^2 - \left(\frac{dt}{d\tau}\right)^2} \qquad u^2 - c^2 = c^2\left(1-\cosh^2\frac{g\tau}{c}\right)$$

$$= c^2\cdot\sinh\frac{g\tau}{c}$$

$$\boxed{\frac{dx}{d\tau} = c\cdot\sinh\frac{g\tau}{c}}$$

$$\boxed{x = \frac{c^2}{g}\left[\cosh\left(\frac{g\tau}{c}\right)-1\right]} \qquad x \simeq \frac{c^2}{g}\left(\cancel{1}+\frac{1}{2}\left(\frac{g\tau}{c}\right)^2-\cancel{1}\right) = \frac{1}{2}g\tau^2$$

$$\boxed{\left(x+\frac{c^2}{g}\right)^2 - c^2t^2 = \frac{c^2}{g}\left(\cosh^2\frac{g\tau}{c} - \sinh^2\frac{g\tau}{c}\right) = \frac{c^2}{g}}$$

hyperbolic motion