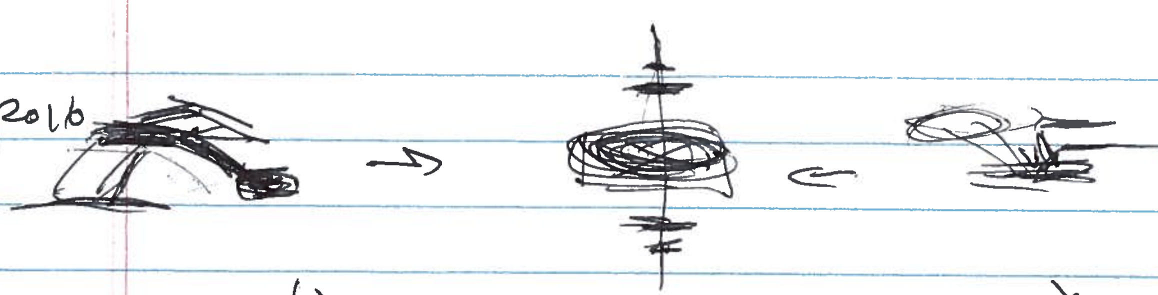
4/1/2016



$u_1 = \begin{pmatrix} \gamma_1 c \\ \gamma_1 \vec{v} \end{pmatrix}$ as seen from (DS9)

$u_2 = \begin{pmatrix} \gamma_2 c \\ \gamma_2 \vec{v} \end{pmatrix}$

as seen from NCC1701 $u_1 = \begin{pmatrix} c \\ 0 \end{pmatrix}$

$u_2 = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix}$

$$u_1 \cdot u_2 = \gamma c \cdot c = \gamma c^2$$

$$u_1 \cdot u_2 = (\gamma_1 c)(\gamma_2 c) - (\gamma_1 \vec{v}_1) \cdot (\gamma_2 \vec{v}_2)$$

$$= \gamma_1 \gamma_2 c^2 \left(1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}\right)$$

$$\boxed{\gamma = \gamma_1 \gamma_2 \left(1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}\right)}$$

colinear, approaching $\rightarrow$ $\gamma_1 \gamma_2 \left(1 + \frac{v_1 v_2}{c^2}\right)$ ✓

$\boxed{\text{more 4-vectors}}$

$\vec{p} = m\vec{v} \longrightarrow p^\mu = m u^\mu$??

$$p^\mu = \begin{pmatrix} \gamma m c \\ \gamma m \vec{v} \end{pmatrix} \longleftarrow \text{relativistic momentum} \quad \longleftarrow ??$$

$$\gamma m c \simeq mc\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = \frac{1}{c}\left(mc^2 + \frac{1}{2}mv^2 + \dots\right)$$

Identify $\boxed{p^0 = \frac{E}{c}}$ $\boxed{E = \gamma m c^2} \rightarrow$ (rest energy) + K.E.

$$p\cdot p = m^2 u\cdot u = m^2c^2$$

$$\left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} = (mc)^2 \qquad \boxed{E^2 = (c\vec{p})^2 + (mc^2)^2}$$

No. $\cancel{m_{rel} = \gamma m = \frac{m}{\sqrt{1-v^2/c^2}}}$

No. $\cancel{ict}$

photon $(m=0)$ $\quad E^2 = c^2|\vec{p}|^2$

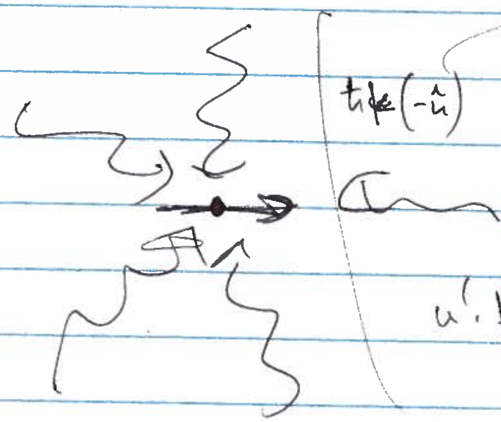
$$E = \hbar\omega, \quad \vec{p} = \hbar\vec{k}, \quad \boxed{p^\mu = \hbar k^\mu} \qquad k^\mu = \begin{pmatrix} \omega/c \\ \vec{k} \end{pmatrix}$$

$$k\cdot x = (k^0)(x^0) - (\vec{k})\cdot(\vec{x}) = \left(\frac{\omega}{c}\right)(ct) - \vec{k}\cdot\vec{x}$$

$$e^{-ik\cdot x} = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

$$\boxed{k^\mu = \begin{pmatrix}\omega/c \\ \vec{k}\end{pmatrix}}$$

phase is frame-invariant.



$\hbar k(-\hat{n})$

$$u_{obs} = \begin{pmatrix}\gamma c \\ \gamma \vec{v}\end{pmatrix}$$

$$u_{lab} = \begin{pmatrix}\gamma c \\ -\gamma \vec{v}\end{pmatrix}$$

$$u' \cdot p : E_0 = \hbar\omega_0 = \begin{pmatrix}\gamma c \cdot \hbar\omega/c \\ +\gamma\vec{v} \cdot \hbar k \hat{n}\end{pmatrix}$$

incoming photon $\quad p^\mu = \frac{E}{c}\begin{pmatrix}1 \\ -\hat{n}\end{pmatrix} = \hbar k \begin{pmatrix}1 \\ -\hat{n}\end{pmatrix}$

moving observer. $\quad u^\mu = \begin{pmatrix}\gamma c \\ \gamma \vec{v}\end{pmatrix}$

$$u\cdot p = (\cancel{c})\left(\frac{\hbar\omega_{obs}}{\cancel{c}}\right) = \left(\frac{\hbar\omega}{\cancel{c}}\right)(\gamma \cancel{c})\left(1 + \frac{\hat{n}\cdot\vec{v}}{c}\right)$$

$$\boxed{\omega_{obs} = \gamma\omega_0\left(1 + \frac{v}{c}\cos\theta\right)} \quad (\text{wrt } \theta_{obs})$$

$$\omega_0 = \gamma\omega\left(1 - \frac{v}{c}\cos\theta\right)$$

$$T_{obs} = \frac{T_0}{\gamma\left(1 + \frac{v}{c}\cos\theta'\right)} \simeq T_0\left(1 + \frac{v}{c}\cos\theta'\right)$$

Electromagnetism

Q = electric charge : (tentatively) scalar.

$$\frac{v}{c} \simeq \frac{1}{137} \cdot 6\cdot 10^{23} \cdot (1.6\times 10^{-19}\,C) \times \left(\frac{1}{\sqrt{1-\frac{1}{137^2}}} - 1 \right) \sim \cancel{1} + \frac{1}{2}\frac{1}{137^2} - \cancel{1}$$

$$= \frac{1.6\times 10^{-19} \cdot 6\cdot 10^{23}}{2\cdot 137^2} = \frac{96{,}000\,C}{37538} = 2.56\,C$$

$$\rho = \frac{dQ}{dV}, \qquad d^4x' = dt'dx'dy'dz' = \left|\det\left(\frac{\partial x'}{\partial x}\right)\right| d^4x$$

$\boxed{\det \Lambda = 1}$ 4-volume is invariant.

$\rho = \dfrac{dQ}{dx\,dy\,dz}$ is time-component of a 4-vector

$J_z = \dfrac{dQ}{dx\,dy\,dt}$ is z-component of a 4-vector

$$d^3\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, dx^\alpha dx^\beta dx^\gamma$$

$\vec{a}\cdot(\vec{b}\times\vec{c})$ = vol.

$$\boxed{J^\mu = \begin{pmatrix} c\rho \\ \vec{J} \end{pmatrix}}$$ 4-vector current density.

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = \frac{1}{c}\frac{\partial}{\partial t}(c\rho) + \vec{\nabla}\cdot\vec{J} = \frac{\partial}{\partial x^0}(J^0) + \frac{\partial}{\partial x^i} J^i$$

$$\boxed{\nabla_\mu J^\mu = 0}$$ equation of continuity is relativistic invariant.

$$\Box^2 = \vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = -\eta^{\alpha\beta}\nabla_\alpha\nabla_\beta$$

$$\Box^2\Phi = (-\rho/\varepsilon_0) \qquad \Box^2\vec{A} = (-\mu_0\vec{J})$$

$$\boxed{A^\mu = \begin{pmatrix}\Phi \\ \vec{A}\end{pmatrix}}$$

But First !

Gaussian Units

2 changes . 1 rule

rule remove μ_0 in favor of $\varepsilon_0 c^2$

$\mu_0\varepsilon_0 = 1/c^2 \qquad \mu_0 = 1/\varepsilon_0 c^2$

change $(c\vec{B})_{SI} \to \vec{B}_{\text{Gaussian}}$

$$\underbrace{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}_{SI} = -\frac{1}{c}\frac{\partial}{\partial t}(c\vec{B}) \qquad \underbrace{\boxed{\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}'}{\partial t}}}_{\text{gaussian}}$$

$\vec{\nabla}\cdot c\vec{B} = 0$ unchanged.

$$\underbrace{\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})}_{SI} = q\left(\vec{E} + \frac{\vec{v}}{c}\times(c\vec{B})\right)$$

$$\boxed{\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}'\right)}$$

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change $\boxed{4\pi\varepsilon_0 = 1}$ $F = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{SI} \frac{Q_1 Q_2}{r^2} = \frac{Q_1}{\sqrt{4\pi\varepsilon_0}} \frac{Q_2}{\sqrt{4\pi\varepsilon_0}} \frac{1}{r^2}$

gaussian: $\boxed{F = \frac{Q_1' Q_2'}{r^2}}$

[coulombs] $\to$ [e.s.u.] $\boxed{e = 4.803 \times 10^{-10} \text{ esu}}$

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \to \boxed{\vec{\nabla}\cdot\vec{E}' = 4\pi\rho}$

$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$= \frac{1}{c^2\varepsilon_0}\vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$

$\to \vec{\nabla}\times(c\vec{B}) = \frac{1}{c\varepsilon_0}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

$\boxed{\vec{\nabla}\times\vec{B}' = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}}$

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_0^2}} = \frac{4\pi}{c}$