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$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \rightarrow \quad \boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \rightarrow \quad \boxed{\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}}$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

above, below (11.141)

Appendix A, Table 2 (p. 781), line 3

$(\vec{D}=\vec{E},\ \vec{H}=\vec{B})$ $\quad \cancel{\varepsilon_0 = 1}$ no. $\quad \boxed{\varepsilon_0 = \frac{1}{4\pi}}$ $\quad$ (1 = Lorentz Heaviside)

$$\boxed{Q^2_{(\text{gaussian})} \longleftrightarrow \frac{Q^2}{4\pi\varepsilon_0}(\text{S.I.})}$$

$\mu_0\varepsilon_0 = \frac{1}{c^2}$, $\varepsilon_0 = \frac{1}{4\pi\cdot 9\cdot 10^9}$, $\mu_0 = 4\pi\cdot 10^{-7}$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

$$= \frac{4\pi}{c}\vec{J} + \frac{1}{c}\left(-\frac{\partial}{\partial t}\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2}\right)$$

$$\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \vec{\nabla}^2\vec{A} = \frac{4\pi}{c}\vec{J} - \vec{\nabla}\left(\frac{1}{c}\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{A}\right)$$

Lorentz gauge: $\boxed{\frac{1}{c}\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0}$ $\quad [\Phi] = [\vec{A}]$

$$\partial_\mu A^\mu = 0$$

11.133

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(-\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = -\vec{\nabla}^2\Phi - \frac{1}{c}\vec{\nabla}\cdot\left(\frac{\partial \vec{A}}{\partial t}\right) = 4\pi\rho.$$

$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \vec{\nabla}^2\Phi = 4\pi\rho \qquad = \frac{4\pi}{c}\cdot c\rho.$$

$$A^\mu = \begin{pmatrix}\Phi \\ \vec{A}\end{pmatrix} \qquad \Box^2 A^\mu = \frac{4\pi}{c} J^\mu \quad (11.133)$$

4-vector potential   vector potential

$\vec{E}, \vec{B}$??   - fourth object? (twice??)
- related to each other?)

note $\quad E^x = -\frac{\partial\Phi}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}(A^x) \qquad (x, t)$

$\quad B^x = \frac{\partial A^z}{\partial y} - \frac{\partial A^y}{\partial z} \qquad (y, z).$

Look at $\quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Antisymmetric. $\quad F^{\mu\nu} = -F^{\nu\mu}$

4×4 : $\quad \frac{4\times3}{2} = 6$ D.f.

$$F^{01} = \nabla^0 A^1 - \nabla^1 A^0 = \overset{+1}{(\eta^{00})}\left(\frac{1}{c}\frac{\partial}{\partial t}\right)(A^x) - \overset{-1}{(\eta^{11})}\left(\frac{\partial}{\partial x}\right)(\Phi)$$

$$= \frac{1}{c}\frac{\partial A^x}{\partial t} + \frac{\partial \Phi}{\partial x} = -(E^x) = -F^{10}$$

et. cyc.

$$F^{23} = \nabla^2 A^3 - \nabla^3 A^2 = (\eta^{22})\left(\frac{\partial}{\partial y}\right)(A^z) - (\eta^{33})\left(\frac{\partial}{\partial z}\right)(A^y)$$

$$= -\frac{\partial A^z}{\partial y} + \frac{\partial A^y}{\partial z} = -(\vec{\nabla}\times\vec{A})^x = -B^x = -F^{32}$$

$$\boxed{F^{\alpha\beta} = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix}} \qquad (11.137)$$

(Always need to work out signs as in this.)

$$F_{\alpha\beta} = \eta_{\alpha\mu}\eta_{\beta\nu}F^{\mu\nu} = (\eta_{\alpha\mu})(F^{\mu\nu})(\eta^T_{\nu\mu}) = \eta F \eta^T$$

$$= \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}(F)\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

each **B** reverses sign twice
each E reverses sign once

$B \to B$
$E \to -E$

(11.138)

Dual (again). $\varepsilon^{\alpha\beta\gamma\delta}$ $\quad \varepsilon^{0123} = +1$

$$*F^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu} = \mathcal{F}^{\alpha\beta}$$

$$*F^{01} = \frac{1}{2}\varepsilon^{01\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(\varepsilon^{0123}F_{23} + \varepsilon^{0132}F_{32}\right)$$

$$= \frac{1}{2}\left((+1)(-B^x) + (-1)(+B_x)\right) = -B^x$$

$$*F^{23} = \frac{1}{2}\varepsilon^{23\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(\varepsilon^{2301}F_{01} + \varepsilon^{2310}F_{10}\right)$$

$$= \frac{1}{2}\left((+1)(E^x) + (-1)(-E^x)\right) = E^x$$

$$*F^{\alpha\beta} = \begin{pmatrix} 0 & -B^x & -B^y & -B^z \\ B^x & 0 & E^z & -E^y \\ B^y & -E^z & 0 & E^x \\ B^z & E^y & -E^x & 0 \end{pmatrix} \quad (11.140)$$

$$\vec{E} \longrightarrow \vec{B}$$
$$\vec{B} \longrightarrow -\vec{E}$$

Dual

Inhomogeneous Maxwell Equations

$\vec{\nabla}\cdot\vec{E} = 4\pi\rho = \frac{4\pi}{c}\cdot c\rho$

$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$

Sum of three derivative of F terms.

E^x, E^y, E^z all in same column.

B^y, B^z, E^x all in same column.

$\nabla_\alpha F^{\alpha\beta}$.

$\nabla_\alpha F^{\alpha 0} = \nabla_0 \cancel{F^{00}} + \nabla_1 F^{10} + \nabla_2 F^{20} + \nabla_3 F^{30}$

$= \nabla_i E^i = 4\pi\rho = \frac{4\pi}{c}\cdot c\rho = \frac{4\pi}{c}\cdot J^0$

$\nabla_\alpha F^{\alpha 1} = \nabla_0 F^{01} + \nabla_1 \cancel{F^{11}} + \nabla_2 F^{21} + \nabla_3 F^{31}$

$= \left(-\frac{1}{c}\frac{\partial}{\partial t}\right)(-E^x) + \left(\frac{\partial}{\partial y}\right)(B^z) + \left(\frac{\partial}{\partial z}\right)(-B^y)$

$= \frac{\partial B^z}{\partial y} - \frac{\partial B^y}{\partial z} - \frac{1}{c}\frac{\partial}{\partial t}E^x = \left(\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t}\right)^x = \frac{4\pi}{c}\vec{J}^x$

$$\boxed{\nabla_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta} \qquad (11.14).$$