4/8/2016 **Jumping Ahead** | Covariant Wave Equation

§12.11

$$\square^2 A^\mu = \frac{4\pi}{c} J^\mu .$$

point particle J^μ

$$\rho = e \cdot \delta^{(3)}(\vec{x} - \vec{r}(t))$$

$$\vec{J} = e\vec{v}\,\delta^{(3)}(\vec{x} - \vec{v}(t))$$

$\vec{v} = \frac{d\vec{r}}{dt}$

$$J^\mu(x) = ec \int d\tau\, U^\mu(\tau)\, \underbrace{\delta^{(4)}(x - r(\tau))}_{\delta(x^0 - r^0(\tau))\,\delta^{(3)}(\vec{x} - \vec{r}(t))} \quad \boxed{12.139}$$

τ^0.

$$\delta(x^0 - r^0(\tau)) = \frac{\delta(\tau - \tau_0)}{\left|\frac{dr}{d\tau}\right|_0} = \frac{1}{\left|\frac{d}{d\tau}(ct)\right|}\,\delta(\tau - \tau^0)$$

$ct = r^0(\tau_0)$

$$= \frac{1}{\gamma c} \cdot \delta(\tau - \tau^0).$$

$$J^\mu(x) = (ec) \cdot \frac{1}{(\cancel{\gamma} c)} \begin{pmatrix} \gamma c \\ \cancel{\gamma}\vec{v} \end{pmatrix} \delta(\vec{x} - \vec{r}(t))$$

$$J^0 = ec \cdot \delta^{(3)}(\vec{x} - \vec{r}(t))$$

$$\vec{J} = e\vec{v}\,\delta^{(3)}(\vec{x} - \vec{r}(t))$$

$$\Box^2 A^\mu = \frac{4\pi}{c} J^\mu$$

$$A^\mu(\vec{x},t) = A^\mu(x) = \int \frac{d^3x'\,dt'}{(c)} \frac{4\pi}{c} J^\mu(x') \frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{x}'|} \delta(t'-t-\tfrac{1}{c}R)$$

$$\delta[(x-x')^2] = \delta\left[(x^0-x'^0)^2 - |\vec{x}-\vec{x}'|^2\right]$$

$$= \delta\left[(x^0-x'^0+R)(x^0-x'^0-R)\right]$$

$$= \frac{1}{2R}\delta(x^0-x^{0'}+R) + \frac{1}{2R}\delta(x^0-x^{0'}-R)$$

$x^{0'} = x^0+R$ adv. $x^{0'} = x^0-R$ ret.

$$G_{ret} = D_r = \frac{1}{2\pi}\delta[(x-x')^2]\,\Theta(x^0-x^{0'}) \qquad ct > ct'$$

$$G_{adv} = D_a = \frac{1}{2\pi}\delta[(x-x')^2]\,\Theta(x^{0'}-x^0) \qquad ct' > ct$$

$$\boxed{A^\mu(x) = \int d^4x'\, D_r(x-x')\, \frac{4\pi}{c} J^\mu(x')} \qquad (12.134)\ (14.1)$$

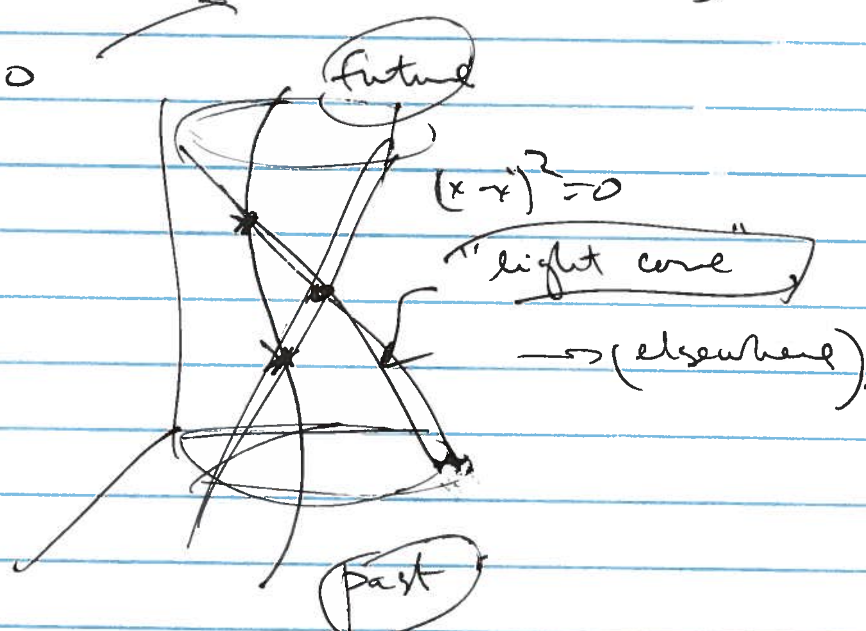
Chapter 14. Radiation by Moving Charges

$$A^\mu(x) = \int \frac{d^4x'}{=} \frac{1}{2\pi} \Theta(x^0 - x'^0)\, \delta[(x-x')^2]$$

$$\cdot ec \cdot \frac{4\pi}{c} \int d\tau\, \underline{V^\mu(\tau)\, \delta^{(4)}(x' - r(\tau))}$$

$$A^\mu = 2e \int d\tau\, V^\mu(\tau)\, \delta[(x - r(\tau))^2]\, \Theta(ct - r^0(\tau))$$

$(x - r(\tau))^2 = 0$



$$\frac{d}{d\tau}[(x - r(\tau))^2] = \frac{d}{d\tau}[(x^0 - r^0(\tau))^2 - (\vec{x} - \vec{r}(\tau))^2]$$

$$= 2(x^0 - r^0)\left(-\frac{dr^0}{d\tau}\right) - 2(\vec{x} - \vec{r}(\tau)) \cdot \left(-2\frac{d\vec{r}}{d\tau}\right)$$

$$= -2(x - r) \cdot \frac{dr}{d\tau} = -2V \cdot (x - r)$$

$$\boxed{A^\mu(x) = \frac{e\, V^\mu(\tau)}{V \cdot (x - \vec{r})}} \quad (14.6)$$

$$x - r = \begin{pmatrix} c(t-t') \\ \vec{R} \end{pmatrix} \qquad V^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{V} \end{pmatrix} = \gamma c \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix}$$

$$\Phi(\vec{x},t) = \frac{e}{R(1-\hat{n}\cdot\vec{\beta})}\Bigg| \qquad \vec{A}(\vec{x},t) = \frac{e\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})}\Bigg|_{t'_{ret}}$$

$t' = t - \frac{1}{c}R$

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Famous $\underbrace{\text{Liénard}}_{(1898)}$ - $\underbrace{\text{Wiechert}}_{(1901)}$ potentials

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad \leftarrow$ differentiate inside integral

$$\partial^\mu A^\nu = 2e \int d\tau\, V^\nu(\tau)\, \Theta(x^0 - r^0(\tau))\, \partial^\mu \delta[(x - r(\tau))^2]$$

$$\partial^\mu \delta[f] = \frac{d}{df}\delta[f] \cdot \partial^\mu f = \partial^\mu f \cdot \frac{d(\delta[f])/d\tau}{df/d\tau}$$

$$f = (x - r(\tau))^2 \qquad \partial^\mu f = 2(x-r)^\mu$$

$$\frac{df}{d\tau} = 2(x-r)\left(-\frac{dr}{d\tau}\right) = -2V\cdot(x-r)$$

$$\partial^\mu A^\nu = 2e \int d\tau\, \frac{d}{d\tau}\left[\frac{(x-r)^\mu V^\nu}{V\cdot(x-r)}\right] \Theta(x^0 - r^0)\, \delta[(x-r)^2]$$

parts

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$$F^{\mu\nu} = \frac{e}{V\cdot(x-r)} \frac{d}{d\tau}\left[\frac{(x-r)^\mu V^\nu - (x-r)^\nu V^\mu}{V\cdot(x-r)}\right]$$

$$(x-r) = R\begin{pmatrix}1\\ \hat{n}\end{pmatrix} \qquad V = \gamma c\begin{pmatrix}1\\ \vec{\beta}\end{pmatrix}$$

$$\frac{dV^\mu}{d\tau} = A^\mu = \begin{pmatrix} c\gamma^4\, \vec{\beta}\cdot\dot{\vec{\beta}} \\ c\gamma^2\dot{\vec{\beta}} + c\gamma^4\vec{\beta}(\vec{\beta}\cdot\dot{\vec{\beta}}) \end{pmatrix}$$

$$\frac{d}{d\tau}\left(V\cdot(x-r)\right) = \underbrace{-V\cdot V}_{-c^2} + (x-r)\cdot\frac{dV}{d\tau}$$

$$\vec{E} = \frac{e(\hat{n}-\vec{\beta})}{\gamma^2(1-\hat{n}\cdot\vec{\beta})^3}\frac{1}{R^2} + \frac{e}{c}\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\hat{n}\cdot\vec{\beta})^3 R}\Bigg|_{t'=t-\frac{1}{c}R}$$

$$\vec{B} = \hat{n}\times\vec{E}\Big|_{t'_{ret}} \qquad (14.14)$$

R^{-2} independent of $\dot{\vec{\beta}}$. Boosted Coulomb field.

R^{-1} $\dot{\vec{\beta}}$ radiation from accelerated q