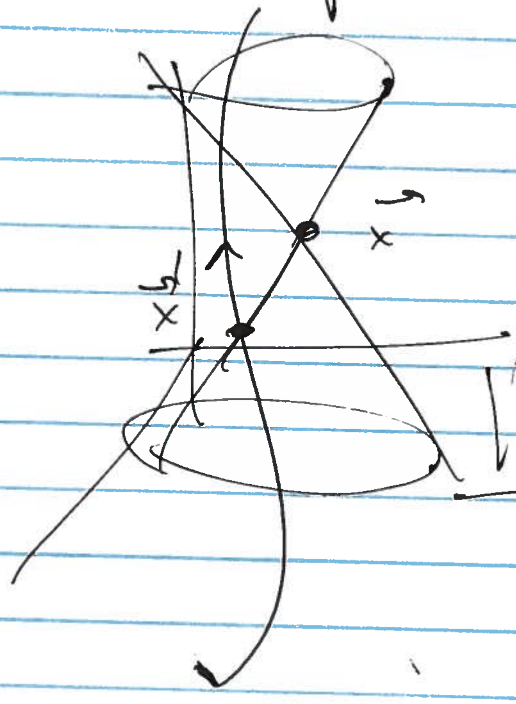
4 | 11 | 2016 Charged particle $r^\mu(\tau) \to \vec{r}(t)$



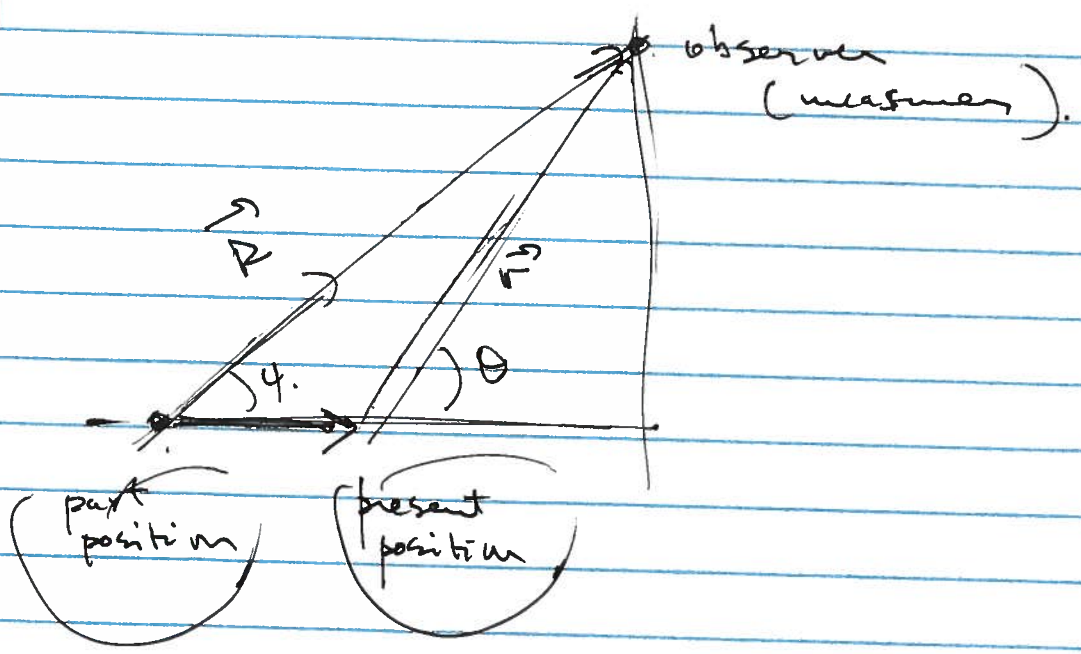
$$|\vec{x}-\vec{r}(t')|^2 = c^2 \Delta t^2 = c^2 (t-t')^2$$

$$\boxed{t'_{ret} = t - \frac{1}{c}|\vec{R}|}$$

$$\vec{E} = \frac{e(\hat{n}-\vec{\beta})}{\gamma^2 (1-\hat{n}\cdot\vec{\beta})^3}\frac{1}{R^2} + \frac{e}{c}\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\hat{n}\cdot\vec{\beta})^3 R}$$

2nd term: radiation.

1st term: $\boxed{\text{Boosted Coulomb field}}$



$$\vec{v}\,\Delta t = \left(\frac{\vec{v}}{c}\right)(c\Delta t) = \vec{\beta} R$$

$$\vec{r} + \vec{\beta} R = \vec{R} \qquad \boxed{\vec{r} = R(\hat{n} - \vec{\beta})}$$

$$\vec{E} = \frac{eR(\hat{n} - \vec{\beta})}{\gamma^2\left[R(1 - \hat{n}\cdot\vec{\beta})\right]^3}$$

$$\left[R(1 - \hat{n}\cdot\vec{\beta})\right]^2 = R^2\left(1 - 2\hat{n}\cdot\vec{\beta} + (\hat{n}\cdot\vec{\beta})^2\right)$$

$$|\vec{r}|^2 = R^2\left(1 - 2\hat{n}\cdot\vec{\beta} + |\beta|^2\right)$$

$$= R^2\left[(1 - \hat{n}\cdot\vec{\beta})^2 + \underbrace{|\beta^2| - |\hat{n}\cdot\vec{\beta}|^2}_{\sin^2\psi}\right]$$

$$\beta^2 R^2 \sin^2\psi = \beta^2 r^2 \sin^2\theta$$

$$\boxed{\vec{E} = \frac{e\vec{r}}{\gamma^2 r^3 (1 - \beta^2 \sin^2\theta)^{3/2}}}$$

✓

Radiation $\vec{E} = \frac{e}{c} \left. \frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{R(1 - \hat{n}\cdot\vec{\beta})^3} \right|_{t'_{ret}}$.

Non-relativistic : $|\vec{\beta}| \ll 1$.

$$\boxed{\vec{E} = \frac{e}{c} \frac{\hat{r} \times (\hat{r} \times \dot{\vec{\beta}})}{r}} \quad \boxed{\vec{B} = \hat{r} \times \vec{E}}$$

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0 c} \vec{E} \times (c\vec{B}) = \underbrace{\frac{c}{4\pi} \vec{E} \times \vec{B}}_{\text{gauss.}}$$

(14.19).

$$= \frac{c}{4\pi} \vec{E} \times \underbrace{(\hat{r} \times \vec{E})}_{\hat{r}(\vec{E}\cdot\vec{E}) - \vec{E}\,\cancel{(\hat{r}\cdot\vec{E})} = \hat{r}|E|^2}$$

$$\boxed{\vec{S} = \frac{c}{4\pi} |\vec{E}|^2 \hat{r}}$$

$$\frac{dP}{d\Omega} = r^2 \hat{r} \cdot \vec{S} = \frac{c}{4\pi} \frac{e^2}{c^2} \left| \hat{r} \times (\hat{r} \times \dot{\vec{\beta}}) \right|^2$$

$$\boxed{\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \left| \hat{r} \times (\hat{r} \times \dot{\vec{\beta}}) \right|^2} \quad (14.20) \quad (*)$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\left|\hat{r}\times(\hat{v}\times\ddot{\vec{x}})\right|^2 = \frac{e^2}{4\pi c^3}\,\dot{v}^2\sin^2\theta$$

$$\int \frac{dP}{d\Omega}\sin\theta\, d\theta\, d\phi \rightarrow \boxed{P = \frac{2}{3}\frac{e^2}{c^3}|\dot{v}|^2} \quad (14.22)$$

$\boxed{\text{Instantaneous}}$

Harmonic.

$$\vec{x} \sim e^{-i\omega t} \qquad \vec{v} = \dot{\vec{x}} = -\omega\vec{x} \qquad \dot{\vec{v}} = -\omega^2\vec{x}$$

$$P = \frac{2}{3}\frac{\omega^4}{c^3}|\vec{p}|^2 \rightarrow \frac{2}{3}\frac{ck^4}{4\pi\varepsilon_0}|\vec{p}|^2 \qquad \vec{p}^*\vec{p}.$$

$4\pi\varepsilon_0$

$$\boxed{\langle P\rangle = \frac{1}{12\pi\varepsilon_0}ck^4|\vec{p}|^2}$$

$$cZ_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\varepsilon_0}. \qquad c^2 Z_0 k^4$$

Relativistic generalization.

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\vec{p}}{dt}\right)\cdot\left(\frac{d\vec{p}}{dt}\right) \qquad \text{3-vector.}$$

$$\longrightarrow \quad -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp}{d\tau}\right)\cdot\left(\frac{dp}{d\tau}\right) \qquad \text{4-vector.}$$

two ways $\quad \frac{dE}{d\tau} = \gamma\frac{dE}{dt} = \gamma(\vec{F}\cdot\vec{v}) = \gamma\left(\frac{d\vec{p}}{dt}\cdot\vec{v}\right) = \vec{v}\cdot\frac{d\vec{p}}{d\tau}$

or. $E^2 = (mc^2)^2 + (c\vec{p})^2 \qquad E\frac{dE}{d\tau} = c^2\vec{p}\cdot\frac{d\vec{p}}{d\tau}$

$$\frac{1}{c}\frac{dE}{d\tau} = \frac{c\vec{p}}{E}\cdot\frac{d\vec{p}}{d\tau} = c\left(\frac{\gamma m\vec{v}}{\gamma mc^2}\right)\cdot\frac{d\vec{p}}{d\tau} = \frac{1}{c}\vec{v}\cdot\frac{d\vec{p}}{d\tau}$$

$$\frac{d\vec{p}}{d\tau} = \frac{d}{d\tau}(\gamma m\vec{v}) = \gamma\frac{d}{dt}(\gamma m\vec{v}) = \gamma^2 m\dot{\vec{v}} + \gamma m\vec{v}\dot{\gamma}$$

$$\dot{\gamma} = \frac{d}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} = -\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\cdot\left(-2\frac{\vec{v}\cdot\dot{\vec{v}}}{c}\right) = \gamma^3\vec{\beta}\cdot\dot{\vec{\beta}}$$

$$\boxed{P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left[\dot{\vec{\beta}}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right]}$$

(Liénard 1898). Lorentz scalar!

Cases: $\dot{\vec{\beta}} \parallel \hat{\beta}$ $\quad \vec{\beta} \times \dot{\vec{\beta}} \to 0$.

linear accelerator, SLAC, ILC.

$$\frac{d\vec{p}}{dt} = \gamma m \dot{\vec{v}} + m\vec{v}\left(\vec{\beta}\cdot\dot{\vec{\beta}}\,\gamma^3\right)$$

$$= \gamma m \dot{\vec{v}} \left(1 + \beta^2\gamma^2\right) = \underline{\underline{\gamma^3 m \dot{\vec{v}}}}$$

$$\left(1 + \frac{\beta^2}{1-\beta^2} = \frac{1-\beta^2-\beta^2}{1-\beta^2} = \gamma^2\right.$$

$$\boxed{P = \frac{2}{3}\frac{e^2}{c^3}\frac{1}{m^2}\left(\frac{d\vec{p}}{dt}\right)^2} \qquad F = \frac{dE}{dx} = \frac{dp}{dt}.$$

$$\boxed{P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2}$$

$$\frac{dE}{dt} = v\frac{dE}{dx} \to \frac{P}{dE/dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\frac{1}{v}\frac{dE}{dx} \to \frac{2}{3}\frac{(e^2/mc^2)}{mc^2}\frac{dE}{dx}.$$

electron: $\quad \frac{mc^2}{(e^2/mc^2)} = \frac{(0.5\,\text{MeV})}{(2.8\,\text{fm})} = \frac{10^6\,\text{eV}}{10^{-15}\,\text{m}} = 10^{21}\,\frac{\text{eV}}{\text{m}}.$

$$\frac{(\text{TeV})}{(\text{km})} = \frac{10^{12}\,\text{eV}}{10^3\,\text{m}} = 10^9\,\frac{\text{eV}}{\text{m}}.$$

parasitic radiation not a problem for ILC.