4/13/2010.

Larmor radiation ($\beta \ll 1$)

$$\vec{E} = \frac{e}{c^2}\,\frac{\hat{r}\times(\hat{r}\times\ddot{\vec{x}})}{r} \qquad \vec{B} = \hat{r}\times\vec{E}$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\left|\hat{r}\times(\hat{r}\times\ddot{\vec{x}})\right|^2$$

$$P = \frac{2}{3}\frac{e^2}{c^3}|\ddot{\vec{x}}|^2$$

instantaneous

$$= \frac{2}{3}\frac{e^2}{m^2c^3}\left|\frac{d\vec{p}}{dt}\right|^2$$

$$\longrightarrow \quad -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp}{d\tau}\right)^2$$

$$\boxed{P = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\left(|\dot{\vec{\beta}}|^2 - |\vec{\beta}\times\dot{\vec{\beta}}|^2\right)}$$

$\vec{\beta}\perp\dot{\vec{\beta}}$ $\quad |\vec{\beta}\times\dot{\vec{\beta}}|^2 = \beta^2\cdot|\dot{\vec{\beta}}|^2$

$$|\dot{\vec{\beta}}|^2(1-\beta^2) = \frac{|\dot{\beta}|^2}{\gamma^2}$$

$\vec{\beta} \perp \dot{\vec{\beta}}$ synchrotron radiation

$$\frac{1}{c}\frac{dE}{dt} \ll \left|\frac{d\vec{p}}{dt}\right| = |\gamma\omega\vec{p}| = \gamma\frac{v}{\rho}p = \gamma\beta\frac{c}{\rho}p$$

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\omega^2p^2 = \frac{2}{3}\frac{e^2}{\cancel{m^2}c^3}\gamma^2\left(\gamma^2\cancel{m^2}\beta^2c^2\right)\left(\frac{\beta^2c^2}{\rho^2}\right)$$

$(\gamma^2 m^2 v^2)$

$$\boxed{P = \frac{2}{3}\frac{e^2c}{\rho^2}\gamma^4\beta^4}$$

$\frac{d}{dt} \to \omega \to \beta$

$p = \gamma m v \to \beta.$

$$\boxed{\delta E/\text{turn} = \frac{2\pi}{\omega}\cdot P = \frac{4\pi}{3}\cdot\frac{e^2}{\rho}\cdot\beta^3\gamma^4}$$

LHC (proton), $2\pi\rho = 27$ km

$\rho = 4.3$ km.

$E = 13$ TeV $mc^2 = 938$ MeV $\gamma \approx 13{,}800$ $\beta \approx 1.$

$$\frac{4\pi}{3}\frac{e^2}{\rho}\gamma^4 = \left(\frac{4\pi}{3}\right)\left(3.3\times10^{-13}\,\text{eV}\right)\left(3.7\times10^{16}\right) = 12\text{ keV}$$

10 keV / proton / turn.

10^{11} / bunch.
~~bunches~~

$\left.\begin{array}{l}10^{11}/\text{bunch} \\ (3)/\text{bunch.} \\ 2808 \text{ "buckets"}\end{array}\right\}$ $10^{14}\cdot 10\,\text{keV}\cdot\left(\frac{27\text{ km}}{\cancel{300{,}000\text{ km}}/s}\right)\left(\frac{300{,}000\text{ km/s}}{30\text{ km}}\right)$

$= 10^{19}$ keV/s $= \boxed{1\text{ kW}}$

Microscopically. Angular distribution

$$\frac{dP}{d\Omega} = r^2 \hat{r} \cdot \vec{S} = \frac{e^2}{4\pi c} \left| \frac{\hat{n} \times ((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}})}{(1 - \hat{n} \cdot \vec{\beta})^3} \right|^2_{t'_r = t - \frac{1}{c} R}$$

rate as observed at $\vec{x}$, emitted at earlier time, travel time depends on distance

$$P'_{\text{source}} = \frac{dE}{dt'} = \frac{dE}{dt} \cdot \frac{dt}{dt'} = P \frac{d}{dt'} \left(t' + \frac{1}{c} (R^2)^{1/2} \right)$$

$$= 1 + \frac{1}{c} \left(\frac{1}{2}\right) (R^2)^{-1/2} \left(2 \vec{R} \cdot \frac{d\vec{R}}{dt'}\right)$$

$$= \underline{1 - \hat{n} \cdot \vec{\beta}}.$$

$$\left(\frac{dP}{d\Omega}\right)_{\text{source}} = \frac{e^2}{4\pi c} \left| \frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}})}{(1 - \hat{n} \cdot \vec{\beta})^5} \right|^2$$

$\dot{\vec{\beta}} \parallel \vec{\beta}$ (linac). $\vec{\beta} \times \dot{\vec{\beta}} = 0$.

$$\frac{dP'}{d\Omega} = \frac{e^2}{4\pi c} \frac{\left|\hat{n} \times (\hat{n} \times \dot{\vec{\beta}})\right|^2}{(1-\hat{n}\cdot\vec{\beta})^5} = \frac{e^2 \dot{v}^2}{4\pi c^3} \cdot \frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

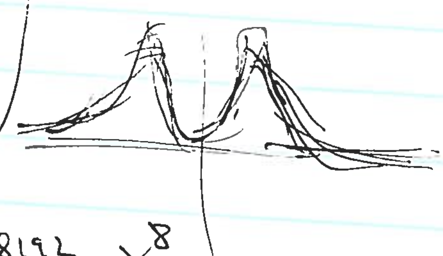
gets big when $\beta \simeq 1$ ($\gamma \gg 1$), $\cos\theta \approx 1$. ($\theta \ll 1$)

$\gamma^2 = \frac{1}{1-\beta^2}$ $\quad \beta^2 = 1 - \frac{1}{\gamma^2}$ $\quad \beta \simeq 1 - \frac{1}{2\gamma^2}$ $\quad \gamma \gg 1$.

$\cos\theta = 1 - \frac{1}{2}\theta^2$

$$\frac{dP'}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{\theta^2}{\left(1 - \left(1 - \frac{1}{2\gamma^2}\right)\left(1 - \frac{1}{2}\theta^2\right)\right)^5}$$

$$\frac{dP'}{d\Omega} \simeq \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{\theta^2}{\left(\frac{1}{2\gamma^2} + \frac{1}{2}\theta^2\right)^5}$$



peaks at $\theta = \frac{1}{2\gamma}$ $\quad$ max $= \frac{8192}{3125} \cdot \gamma^8$

$$\int d\Omega \cdot \frac{dP'}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \cdot 2\pi \underbrace{\int_0^{(\pi)} \theta\, d\theta \frac{\theta^2}{\left(\frac{1}{2\gamma^2} + \frac{1}{2}\theta^2\right)^5}}_{\frac{4}{3}\gamma^6}$$

$$P' = \frac{e^2 \dot{v}^2}{4\pi c^3} \cdot 2\pi \cdot \frac{4}{3}\gamma^6 = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3} \cdot \gamma^6 \quad \checkmark!$$

(5)

$\dot{\vec{\beta}} \perp \vec{\beta}$ let $\vec{\beta} = \beta\hat{z}$

$\dot{\vec{\beta}} = -\dot{\beta}\hat{x}$

$$\frac{dP'}{d\Omega} = \frac{e^2\dot{v}^2}{4\pi c^3} \frac{1}{(1-\beta\cos\theta)^3}\left[1 - \frac{\sin^2\theta \cdot \cos^2\phi}{\gamma^2(1-\beta\cos^2\theta)^2}\right]$$

$$\simeq \frac{e^2\dot{v}^2}{4\pi c^3} \frac{1}{(\frac{1}{2}\gamma^2+\frac{1}{2}\theta^2)^3}\left[1 - \frac{(\theta/\gamma)^2 \cos^2\phi}{(\frac{1}{2}\gamma^2+\frac{1}{2}\theta^2)^2}\right]$$

peaks (again) $\theta \sim \pm\frac{1}{\gamma}$

$\int d\Omega \longrightarrow$ $P' = \frac{2}{3}\frac{e^2\dot{v}^2}{c^3}\cdot\gamma^4$