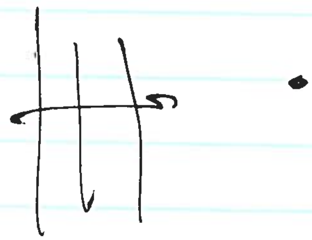
4/15/2016 Thomson scattering (PhD Tree)



$$\vec{E} = E_0 \hat{\varepsilon}_0 e^{i(\vec{k}_0\cdot\vec{x} - \omega t)}$$

$$m\ddot{\vec{x}} = m\dot{\vec{v}} = e\vec{E}$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\left|\hat{r}\times(\hat{r}\times\ddot{\vec{x}})\right|^2 = \frac{e^2}{4\pi c^3}\left|\hat{r}\times\left(\hat{r}\times\frac{e\vec{E}}{m}\right)\right|^2$$

$$\vec{S}_{in} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\vec{E}_0\times(\hat{k}_0\times\vec{E}) = \frac{c}{4\pi}\hat{k}_0|\vec{E}_0|^2$$

$$\langle\cdot\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}^*\cdot\vec{E}) \qquad (\mathrm{J.\ Re}).$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{e^2}{4\pi c^3}\frac{e^2}{m^2}\left|\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\right|^2 \cancel{|E_0|^2}}{\frac{c}{4\pi}\cancel{|E_0|^2}}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2\left|\hat{r}\times(\hat{r}\times\vec{\varepsilon}_0)\right|^2}$$

$$|\hat{r}\times\vec{\varepsilon}_0|^2 = (\hat{r}\times\vec{\varepsilon}_0)^*\cdot(\hat{r}\times\vec{\varepsilon}_0) = \hat{r}\cdot\left(\vec{\varepsilon}_0^*\times(\hat{r}\times\vec{\varepsilon}_0)\right)$$

$$= \hat{r}\cdot\left[\hat{r}(\vec{\varepsilon}_0^*\cdot\vec{\varepsilon}_0) - \vec{\varepsilon}_0(\hat{r}\cdot\vec{\varepsilon}_0)^*\right] = 1 - (\hat{r}\cdot\vec{\varepsilon}_0)^2$$

$E^{\parallel}$ $E^{\perp}$

$\hat{r}\times(\hat{r}\times\hat{\varepsilon}_0)\cdot\hat{\theta} = -\cos\theta\cdot\cos(\phi-\phi_0)$

$\hat{r}\times(\hat{r}\times\hat{\varepsilon}_0)\cdot\hat{\phi} = \sin(\phi-\phi_0)$

polarized

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

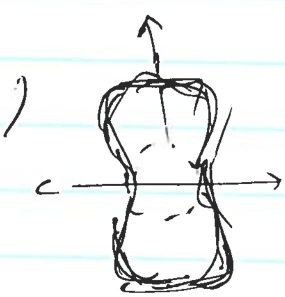
$$\hat{\varepsilon}_0 = \hat{x}\cos\phi_0 + \hat{y}\sin\phi_0$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2\left[1 - \sin^2\theta\cos^2(\phi-\phi_0)\right]}$$ $\theta = \frac{\pi}{2}$

unpolarized incident $\rightarrow$ $\langle\cos^2(\phi-\phi_0)\rangle = \frac{1}{2}$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2}(1+\cos^2\theta)} \quad (14.125)$$

$$\boxed{\sigma_T = \frac{8\pi}{3}\cdot\left(\frac{e^2}{mc^2}\right)^2} \quad (14.126)$$



J.J. Thomson. 1906 Nobel prize.

$$r_0 = \frac{e^2}{mc^2} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{mc^2} = 2.8\times10^{-15}\ \text{m}.$$ fm.

$$\sigma_T = 6.65\times10^{-25}\ \text{cm}^2 = 0.665\ \text{“barn”}$$

QFT. $\hbar\omega \ll mc^2 \rightarrow \left(1 - 2\frac{\hbar\omega}{mc^2}\right)$ Compton

$\hbar\omega \gg mc^2 \rightarrow \frac{3}{4}\frac{mc^2}{\hbar\omega}\log\left(\frac{2\hbar\omega}{mc^2}\right)$ Klein-Nishina

CMB. opaque until (380,000 y).

$$P_{scatt} = \left(\frac{dE}{dt}\right)_{scatt} = \left(\frac{dP}{dA}\right)_{wave} \cdot \sigma_T = \frac{L}{4\pi R^2} \cdot \sigma_T$$

$\underline{E = \hbar\omega}$ $\underline{\vec{p} = \hbar\vec{k}}$ $\left(\frac{\vec{g}}{u} = \frac{\vec{E}\times\vec{B}/4\pi c}{\frac{1}{8\pi}(E^2+B^2)} = \frac{\vec{k}}{\omega} \right)$

$$\left(\frac{d\vec{p}}{dt}\right)_{scatt} = \left(\frac{\vec{k}}{\omega}\right)\left(\frac{dE}{dt}\right)_{scatt} .$$

(incident . coherent
out . symmetric 4π +/−

$\frac{\vec{k}}{\omega} = \frac{\hat{r}}{c}$. $\boxed{\vec{F}_{rad} = \hat{r} \cdot \frac{L}{4\pi R^2} \cdot \frac{\sigma_T}{c}}$

Balanced against. $\vec{F}_{grav} = -\frac{GM}{R^2} m_{(p)} \hat{r}$

$$\frac{GM}{R^2} m_p = \frac{L}{4\pi R^2} \frac{\sigma_T}{c}$$

$$L < \boxed{L_E = \frac{4\pi GM m_p c}{\sigma_T}}$$

Sun: $M_\odot = 2\times10^{33}\,g$

$L_\odot = 4\times10^{33}\,erg\,s^{-1}$

$L_E = \frac{4\pi G M m_p c}{\sigma_T} = 1.26\times10^{38}\,erg\,s^{-1}$

$L/L_E \simeq 3\times10^{-5}$. Falls along main sequence

O*, $M > 16 M_\odot$

"Wolf-Rayet". $M > 20 M_\odot$ T. 200,000 K

Lifetime of a star,

$\Delta E = \varepsilon M c^2 = 0.007\, Mc^2$ $\left(\frac{28\,MeV}{4\,GeV} = 0.007\right)$

$L = \gamma L_E$.

$$\tau = \frac{\varepsilon \cancel{M} c^2}{\gamma} \frac{1}{4\pi G \cancel{M} m_p c/\sigma_T} = \frac{\varepsilon}{\gamma}\cdot\frac{8\pi}{3}\frac{(e^2/m_e c^2)^2\, c}{4\pi G m_p}$$

$$= \frac{2}{3}\frac{\varepsilon}{\gamma}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{\hbar c}{G m_p m_e}\right)\left(\frac{\hbar}{m_e c^2}\right)$$

$$= \frac{2}{3}\frac{\varepsilon}{\gamma}\left(\frac{1}{137.036}\right)^2\left(\underset{3.11}{3\times10^{41}}\right)\left(1.3\times10^{-21}\,s\right)$$

$$= \frac{\varepsilon}{\gamma}\left(4.5\times10^{8}\,year\right) = \frac{3\,M\,year}{(\gamma)}$$

(Sun → 100 Gy)

Back to chapter 12.

Classical physics $S = \int dt\, L(q_j, \dot{q}_j)$

$$L = \sum_j \tfrac{1}{2}\dot{q}_j^2 - V.$$

$q_j(t) \to \phi(\vec{x}, t)$ $\qquad \sum_j \to \int d^3x.$

$$S = \int dt \int d^3x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \tfrac{1}{2}\dot{\phi}^2 - \tfrac{1}{2}|\vec{\nabla}\phi|^2 - V(\phi) = \tfrac{1}{2}\partial_\alpha\phi\, \partial_\beta\phi\, g^{\alpha\beta}.$$

$$\delta S = S[\phi + \delta\phi] - S[\phi]$$

$$= \int d^4x \left[\frac{\partial \mathcal{L}}{\partial \phi} \cdot \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \cdot \delta(\partial_\mu \phi) \right]$$

$$\delta(\partial_\mu \phi) = \partial_\mu(\phi + \delta\phi) - \partial_\mu \phi = \partial_\mu(\delta\phi).$$

$$\delta S = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial \phi} \cdot \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \cdot \partial_\mu(\delta\phi) \right]$$

parts

$$= \int d^4x \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \right] \cdot \delta\phi = 0$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

$$\mathcal{L} = \frac{1}{2} (\partial_\alpha \phi)(\partial_\beta \phi) \eta^{\alpha\beta} - V$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{1}{2} (\partial_\beta \phi) \eta^{\mu\beta} + \frac{1}{2} (\partial_\alpha \phi) \eta^{\alpha\mu} = \underline{\partial^\mu \phi}.$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = - \frac{\partial V}{\partial \phi}.$$

$$\partial_\mu (\partial^\mu \phi) = \boxed{\Box^2 \phi = - \frac{\partial V}{\partial \phi}}$$

$$V = \frac{1}{2} m^2 \phi^2 \qquad \boxed{\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = - m^2 \phi}$$

$$e^{i(\vec{k} \cdot \vec{x} - \omega t)} \qquad -\omega^2 + \vec{k}^2 = - m^2$$

$$\boxed{(\hbar \omega)^2 = (\hbar c \vec{k})^2 + (mc^2)^2}$$