PHZ 6607 Fall 2016

Midterm Exam, Due 6:00pm, Friday October 21

1. A photon with frequency ω moving in the z-direction at a distance b off the z-axis bounces off a perfectly reflecting sphere of radius a located at the origin.

(a) The sphere sits at rest. What is the angle between the final and initial photon momentum? What is the frequency of the reflected photon? What is the differential scattering cross section? What is the total cross section?

(b) Now, the reflecting sphere is moving in the $+z$-direction with speed v. What are the answers to the questions in part (a)? (Do not assume $v \ll 1$.)

(c) The sphere sits in an isotropic radiation bath with incoming flux $\mathcal{F}_0$ (number per area per steradian per time) in all directions. What is the force on the sphere at rest? What is the force on the sphere moving at speed v?

2. The flux of high energy cosmic rays is expected to fall off rapidly above the energy threshold for pion photoproduction when a very high energy cosmic ray proton ($E \gg m_p = 938\,\mathrm{MeV}$) scatters with a 3 K microwave background photon ($E_\gamma = 2.3 \times 10^{-4}\,\mathrm{eV}$) and (through an intermediate Δ^+) creates a neutral pion ($m_\pi = 135\,\mathrm{MeV}$), $p + \gamma \to p + \pi$.

(a) What is the minimum energy the proton must have in order to create a pion at rest with respect to the proton, as a function of the angle between $\boldsymbol{p}_p$ and $\boldsymbol{p}_\gamma$?

(b) What is the energy of the proton after the collision? Does it lose an appreciable amount of energy? What is its momentum? What is its velocity? Has it slowed down?

3. Let $M^{\alpha\beta\gamma} = x^\alpha T^{\beta\gamma} - x^\beta T^{\alpha\gamma}$ and $M^{\alpha\beta} = \int d^3x\, M^{\alpha\beta 0}$.

(a) Is $M^{\alpha\beta}$ a conserved quantity? What is $dM^{\alpha\beta}/dt$?

(b) Let X^i be the "center of mass" coordinate, defined by

$$X^i = \frac{\int d^3x\, x^i\, T^{00}}{\int d^3x\, T^{00}}.$$

What is dX^i/dt?

(c) Let $M^{0i} = K^i$ and $M^{ij} = \epsilon^{ijk} J_k$. In terms of $\boldsymbol{J}$ and $\boldsymbol{K}$, what is $M^{\alpha\beta} M_{\alpha\beta}$? What is the effect of a boost of speed v along the x-axis on $\boldsymbol{J}$ and $\boldsymbol{K}$? What is $M^{\alpha\beta} M_{\alpha\beta}$ after the boost?

4. Let x, y, z be the usual Cartesian coordinates in flat space. Oblate spheroidal coordinates ξ, α, β are defined by the relations

$$x = \sqrt{\xi^2 + c^2}\,\sin\alpha\,\cos\beta, \qquad y = \sqrt{\xi^2 + c^2}\,\sin\alpha\,\sin\beta, \qquad z = \xi\,\cos\alpha,$$

where c is a constant.

(a) What is the shape of surfaces of constant ξ?

(b) What is the metric in ξ, α, β?

(c) What is the Laplacian operator on a scalar field, $\nabla^2\Phi$, in these coordinates?

(d) Show that $\nabla^2\Phi = 0$ is separable in oblate spheroidal coordinates, so that the potential can be written $\Phi(\xi, \alpha, \beta) = X(\xi)\,A(\alpha)\,B(\beta)$. Find the $A(\alpha)$ and $B(\beta)$ solutions explicitly, and write an equation for $X(\xi)$.

$(*)$ If $\xi = \xi_0$ is the surface of an oblate conducting surface that carries net charge Q, what is the electrostatic potential exterior to ξ_0? What is the surface charge density as a function of $\rho = \sqrt{x^2 + y^2}$ in the limit that the spheroid becomes a flat disk, $\xi_0 \to 0$? What is the capacitance of the disk?

5. If two metrics satisfy

$$g'_{\mu\nu} = e^{2\Omega(x)}\,g_{\mu\nu}$$

then they are said to be related by a conformal transformation.

(a) Show that a conformal transformation preserves angles.

(b) If g has connection coefficients Γ, what are the connection coefficients Γ' for g'?

(c) Show that a conformal transformation preserves null geodesics.