PHY 6607 Fall 2016

Homework #1, Due Friday, September 2

Newtonian gravity makes its own predictions for some phenomena held up as successes of General Relativity. For later comparison, let's compute some of these Newtonian results.

1. What is the escape velocity for a particle of mass m at a distance R from a mass M? At what value of R does the escape velocity become c? What is this for the sun ($M_\odot = 1.99 \times 10^{33}\,\mathrm{g}$)?

2. Motion in a Newtonian gravitational potential is independent of mass, and so gravity plausibly affects even massless particles (photons) moving at the speed of light. Suppose a particle of mass m heads towards an object of mass M with initial speed $v = c$ and impact parameter b far from M. By what angle is the particle deflected? (Take the deflection angle to be small.) Compute the numerical value of the deflection angle for the path of a ray of light from a distant star that just grazes the surface of the sun ($r_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$). Express your answer in arcseconds.

3. Tides occur because the force is gravity is slightly different at two nearby points.

(a) What is the difference between the gravitational acceleration induced by a mass M for points separated by a small displacement $\boldsymbol{h}$ at a distance R from M (take $h \ll R$). (The answer is a function of angle as well as distance). How many high/low tides are there in a day?

(b) If the earth were a perfect sphere covered with water, estimate the height difference between high and low tides (ignoring complications such as resonance, rotation, friction, viscosity) for spring tides (directions of sun and moon aligned) and neap tides (sun and moon at right angles). [Potentially useful data: $M_\oplus = 5.98 \times 10^{27}\,\mathrm{g}$, $r_\oplus = 6471\,\mathrm{km}$, $M_☾ = 7.35 \times 10^{25}\,\mathrm{g}$, $R_\odot = 1.50 \times 10^{13}\,\mathrm{cm}$, $R_☾ = 384,000\,\mathrm{km}$]

(c) A neutron star is a collapsed object of nuclear density with mass $M_\star = 1.4\,M_\odot$, radius $r_\star = 10\,\mathrm{km}$. In Larry Niven's short story *Neutron Star* (1966), tidal forces in the neighborhood of the title object prove fatal to the unwary. What is likely to be the orientation of a needle-like spaceship at its closest approach at a distance of 20 km from the center of a neutron star? What is the force experienced by the unfortunate who finds himself at one of the ends of a 100 m spaceship? What is the tidal acceleration difference across the diameter of a person (say a distance of 1 m)? Is it likely that the pilot can hold himself close enough to the center-of-mass of the spaceship to avoid injury?

4. One effect of General Relativity in the solar system is an otherwise unexplained excess precession of the orbit of Mercury, by 43 seconds of arc per century. However, this is only a small part of the observed precession, mostly due to the gravitational influence of Jupiter and other planets. Compute, or estimate, the Newtonian rate of precession of Mercury's orbit caused by Jupiter (note that the statement of this question is deceptively simple). [More useful data: the orbital period of Mercury is $T_{☿} = 87.97\,\mathrm{d}$; masses and orbital radii are $M_{☿} = 0.0554$, $M_{♃} = 317.83$, $R_{☿} = 0.387$, $R_{♃} = 5.203$ (relative to Earth).]