PHZ 6607 Fall 2016

Homework #5, Due Friday, October 14

1. Let ∇_u be the directional derivative $u^\alpha \nabla_\alpha$. Let $[u,v] = \nabla_u v - \nabla_v u$.

(a) Show that without torsion $\nabla_u \nabla_v \Phi - \nabla_v \nabla_u \Phi = \nabla_{[u,v]} \Phi$, where Φ is a scalar function.

(b) If u and v are coordinate directions, what is $\nabla_\alpha \nabla_\beta \Phi - \nabla_\beta \nabla_\alpha \Phi$?

2. Let $g_{\alpha\beta}$ in a coordinate basis be diagonal with elements g_α. Let $g = \det g_{\alpha\beta}$. Show that

$$\nabla^2 \Phi = \nabla_\alpha \nabla^\alpha \Phi = \Phi^{;\alpha}{}_{;\alpha} = \frac{1}{\sqrt{|g|}} \left(\sqrt{|g|}\, g^{\alpha\beta}\, \Phi_{,\beta} \right)_{,\alpha} .$$