1. Let $\nabla u$ be the directional derivative $u^\alpha \nabla_\alpha$. Let $[u, v] = \nabla_u v - \nabla_v u$.

(a) Show that without torsion $\nabla_u \nabla_v \Phi - \nabla_v \nabla_u \Phi = \nabla_{[u,v]} \Phi$, where $\Phi$ is a scalar function.

(b) If $u$ and $v$ are coordinate directions, what is $\nabla_\alpha \nabla_\beta \Phi - \nabla_\beta \nabla_\alpha \Phi$?

2. Let $g_{\alpha\beta}$ in a coordinate basis be diagonal with elements $g_\alpha$. Let $g = \det g_{\alpha\beta}$. Show that

$$\nabla^2 \Phi = \nabla_\alpha \nabla^\alpha \Phi = \Phi^{\alpha};_\alpha = \frac{1}{\sqrt{|g|}} \left( \sqrt{|g|} g^{\alpha\beta} \Phi^{;_\beta}_\alpha \right),_\alpha.$$. 