8/22/16

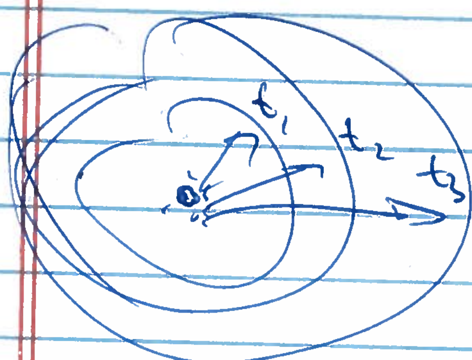
$x = x^\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$ 4-vector $\mu = 0,1,2,3$

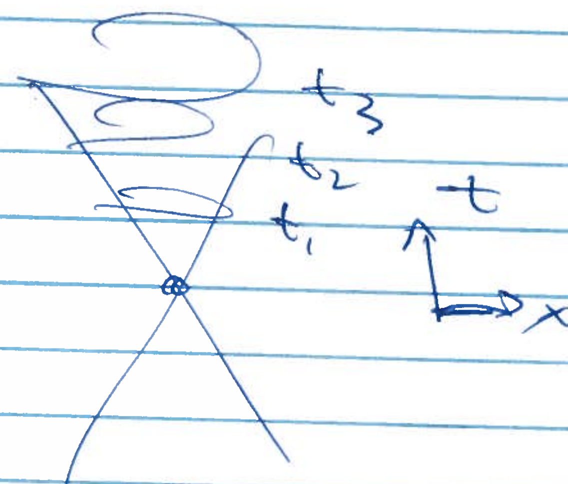
$\vec{x} = x^i$ $i = 1,2,3$

x^a x^A

AE, PSCP, JDJ $j \leftrightarrow \alpha\beta$

$\boxed{c = 1}$ $(\hbar = 1)$.





Observer O. $(\Delta \vec{x})^2 = c^2(\Delta t^2) = \Delta t^2$.

Observer O'. $(\Delta x')^2 = (\Delta t')^2$

$\Delta s^2 = -(\Delta t)^2 + (\Delta \vec{x})^2$ invariant

$x \cdot y = -(x^0)(y^0) + (x^1)(y^1) + (x^2)(y^2) + (x^3)(y^3)$

$= -x^0 y^0 + \vec{x} \cdot \vec{y} = \eta_{\mu\nu} x^\mu y^\nu$

$\eta = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

1 superscript / 1 subscript / $\sum$ (different)

opposite signature

②

$g = 980\ \mathrm{cm\,s^{-2}} \longrightarrow \sqrt{\text{temperature}}$

$\hbar g = (10^{-27}\ \mathrm{erg\text{-}s})(10^{3}\ \mathrm{cm\,s^{-2}}) = 10^{-24}\ \mathrm{erg\cdot cm\,s^{-1}}$

$\frac{\hbar g}{c} = 3\times 10^{-35}\ \mathrm{erg} = 2\times 10^{-23}\ \mathrm{eV} = 2\cdot 10^{-19}\ \mathrm{K}$

$(1\ \mathrm{eV} = 1.602\times 10^{-12}\ \mathrm{erg},\quad 1\ \mathrm{eV} = 11{,}604.5\ \mathrm{K})$

$[K] = [\mathrm{cm\,s^{-2}}][c]^a[\hbar]^b[G]^c[k]^d$

$= [\mathrm{cm\,s^{-2}}][\mathrm{cm\,s^{-1}}]^a[\mathrm{erg\cdot s}]^b[\mathrm{cm^3\,erg^{-1}\,s^{-2}}]^c[\mathrm{erg\,K^{-1}}]^d$

cm $\quad 0 = 1 + a + 3c$

s $\quad 0 = -2 - a + b - 2c$

erg $\quad 0 = 0 + b - c + d.$

K $\quad \boxed{1 = -d} \longrightarrow d = -1$

$1 + a + 3c = 0$

$-2 - a + b - 2c = 0$

$-b + c - d = 0$

$2c = 0 \quad c = 0$

$a = -1 \quad b = 1.$

$\frac{\hbar g}{k_B c} = 2.5\times 10^{-19}\ \mathrm{K}$

3

$\vec{F} = m\vec{a}$

$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

$$\frac{d^2\vec{x}}{dt^2} = -\vec{\nabla}\phi \quad \Longleftrightarrow \quad \frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} \qquad (4.6)$$

$$\frac{d^2x^i}{dt^2} = -\frac{\partial\phi}{\partial x_i} = \phi_{,i}$$

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\gamma}\left(g_{\gamma\alpha,\beta} + g_{\gamma\beta,\alpha} - g_{\alpha\beta,\gamma}\right)$$

(3.27)

$d\tau^2 = -ds^2$

$= dt^2 - d\vec{x}^2$

(coordinate basis).

"geodesic equation" $\nabla_u u = 0$.

Newton

$$\nabla^2\phi = 4\pi G\rho$$

$$T = \begin{pmatrix} \rho & & \\ & p & \\ & & p \end{pmatrix}$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$G = \partial\Gamma + \Gamma\Gamma$

Einstein

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \qquad (1.5)$$

$$R^\alpha{}_{\beta\mu\nu} = \Gamma^\alpha_{\nu\beta,\mu} - \Gamma^\alpha_{\mu\beta,\nu} + \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\beta} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$$

(3.113)

"world line" $x^\mu(\lambda)$

λ arbitrary parameter $\left(\frac{dt}{d\lambda} > 0\right)$

$$d\tau^2 = -dx \cdot dx = dt^2 - d\vec{x}^2$$

$$= dt^2\left(1 - \left(\frac{d\vec{x}}{dt}\right)^2\right) = dt^2(1-v^2)$$

$d\tau$ is time interval in frame $d\vec{x} = 0$

= frame comoving with (observer) = proper frame

$$\boxed{d\tau = \text{"proper time"}}$$

$d\tau \leq dt$ ("moving clocks run slow")

$$\gamma = \frac{1}{\sqrt{1-v^2}} \qquad d\tau^2 = (1-v^2)dt^2 \qquad \boxed{d\tau = \frac{dt}{\gamma}}$$

parametrize with τ, $x^\mu(\tau)$

tangent: $u^\mu = \frac{dx^\mu}{d\tau}$

$$u \cdot u = \frac{dx}{d\tau} \cdot \frac{dx}{d\tau} = -\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{d\vec{x}}{d\tau}\right)^2$$

$$= -\frac{(+dt^2 - d\vec{x}^2)}{d\tau^2} = -\left(\frac{d\tau}{d\tau}\right)^2 = -1$$