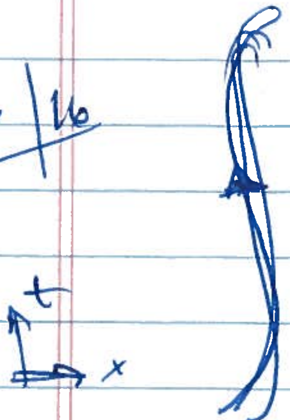
8/24/16



1-d curves: $x^\mu = x^\mu(\lambda)$

λ arbitrary $\left(\frac{dt}{d\lambda} \geq 0\right)$ tangent $u^\mu = \frac{dx^\mu}{d\lambda}$

standard normalization?

(1.17)

timelike. $d\tau^2 = -ds^2 = -dx \cdot dx = dt^2 - (d\vec{x})^2$

$= dt^2\left(1 - \left(\frac{d\vec{x}}{dt}\right)^2\right) = dt^2(1-v^2)$

$d\tau$ is time interval in frame where $d\vec{x} = 0$

= frame comoving with observer = proper frame

$d\tau$ = "proper time"

$1 - v^2 < 1 \Rightarrow d\tau < dt$. "moving clocks run slow"

$\gamma = \frac{1}{\sqrt{1-v^2}}$ $\quad d\tau = \frac{dt}{\gamma}$ $\quad \frac{d}{d\tau} = \gamma \frac{d}{dt}$.

parametrize with τ. $x^\mu(\tau)$

$u^\mu = \frac{dx^\mu}{d\tau}$ $\quad u \cdot u = \frac{dx}{d\tau} \cdot \frac{dx}{d\tau} = -\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{d\vec{x}}{d\tau}\right)^2$

$= -\gamma^2 + \gamma^2 v^2 = -1$

$u^\mu = \begin{pmatrix} \frac{dt}{d\tau} \\ \frac{d\vec{x}}{d\tau} \end{pmatrix} \begin{pmatrix} \gamma \\ \gamma\vec{v} \end{pmatrix}$

J|V

Prox. Cen

Eclipse 8/21/17

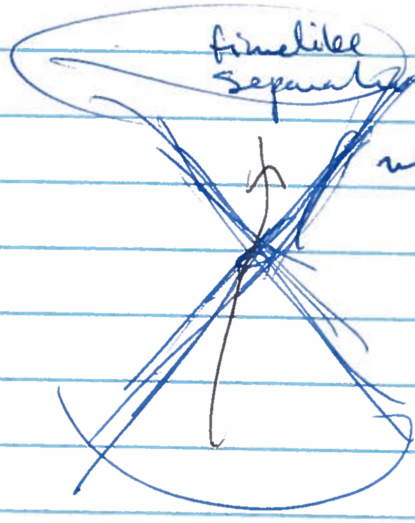
SN

oregon – SC

u^μ is timelike unit tangent,

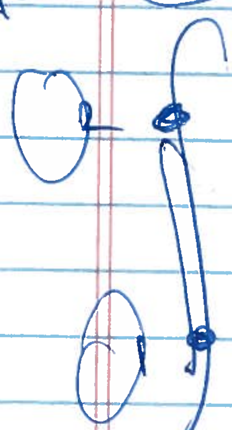
spacelike, $ds^2 = dx \cdot dx$ $(-d\tau^2)$,

$x^\mu = x^\mu(s)$ $\frac{dx^\mu}{ds} = n^\mu$ $n \cdot n = 1$.



divided by "light cone"

two events



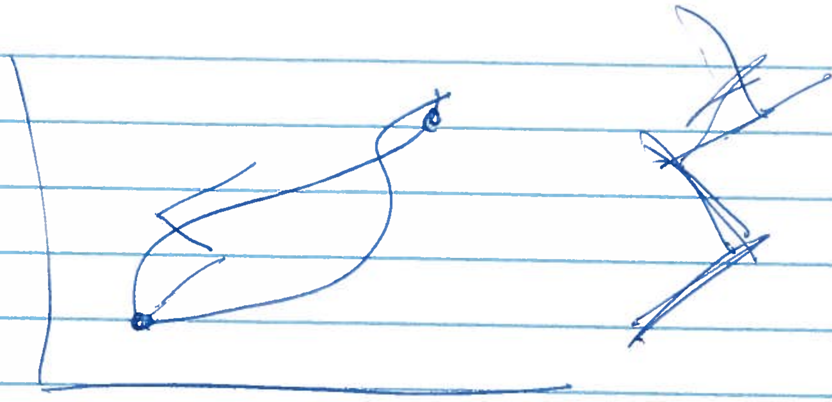
$$\Delta\tau_{12} = \int_1^2 d\lambda \left[\left(\frac{dt}{d\lambda}\right)^2 - \left(\frac{d\vec{x}}{d\lambda}\right)^2\right]^{1/2}$$

(1.22).

depends on trajectory.

"functional" of path.

3



timelike. can always get from ① to ② by sequence of (45°) segments. (null).

$\longrightarrow (\Delta\tau = 0)$

Twin Paradox

wel.

$t' = \gamma(t - vx)$

$x' = \gamma(x - vt)$

$\Delta\tau \Rightarrow$ action. $S = (-m)\int d\lambda \cdot \left(-\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}\eta_{\alpha\beta}\right)^{\frac{1}{2}}$

$\delta S = 0 \rightarrow \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial x'^\mu}\right) - \cancel{\frac{\partial \mathcal{L}}{\partial x^\mu}} = 0$

$\frac{d}{d\lambda}\left(\frac{1}{2}\right)\left(\left(-\frac{dx}{d\lambda}\right)^2\right)^{\frac{1}{2}}\left(-2\,\eta_{\alpha\mu}\frac{dx^\alpha}{d\lambda}\right)$

take $\lambda = \tau$ $(\lambda = a\tau + b) \Longleftrightarrow \left(\frac{dx}{d\tau}\right)^2 = -1$

$\frac{d}{d\tau}\left(\eta_{\alpha\mu}\frac{dx^\alpha}{d\tau}\right) = 0$ $\left(\frac{d^2x^\alpha}{d\tau^2} = 0\right)\left(\frac{du^\alpha}{d\tau} = 0\right)$

√ straight line. (~~unaccelerated~~) unaccelerated

$u^\mu = \frac{dx^\mu}{d\tau}$ 4-velocity $\boxed{u \cdot u = -1}$

$a^\mu = \frac{du^\mu}{d\tau}$ 4-acceleration.

$\frac{d}{d\tau}(u \cdot u) = \frac{d}{d\tau}(-1) = 0 = 2u \cdot a$ $\boxed{u \cdot a = 0}$

rest frame (proper frame) $a^\mu = \begin{pmatrix} 0 \\ \vec{a} \end{pmatrix}$

$a \cdot a = a^2 \underline{> 0}$ (spacelike)
proper acceleration

$\boxed{p^\mu = m u^\mu}$ $p^0 = \gamma m = \frac{m}{\sqrt{1-v^2}} = m + \frac{1}{2} m v^2 + \cdots$
(rest energy) (kinetic energy)

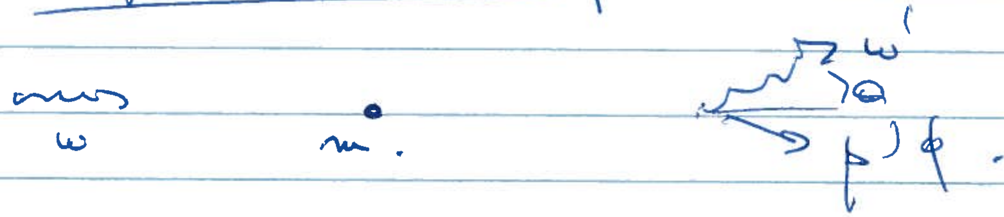
$p^2 = p \cdot p = m^2 = E^2 - |\vec{p}|^2$

$$\boxed{E^2 = |\vec{p}|^2 + m^2}$$

$$\boxed{\frac{\vec{p}}{E} = \frac{\gamma m \vec{v}}{\gamma m} = \vec{v}}$$

$$\left(\sum p^\mu\right)_{before} = \left(\sum p^\mu\right)_{after}$$

Compton scattering.



$$E: \ \omega + m = \omega' + E$$

$$p_x: \ k = k'\cos\theta + p\cos\phi$$

$$p_y = 0 = k'\sin\theta - p\sin\phi$$

$$(k - k'\cos\theta)^2 = p^2\cos^2\phi = p^2 - p^2\sin^2\phi = p^2 - \omega'^2\sin^2\theta$$

$$k^2 + k'^2\cos^2\theta - 2kk'\cos\theta = p^2 - k'^2\sin^2\theta$$

$$\boxed{k^2 + k'^2 - 2kk'\cos\theta = p^2}$$

$$(\omega + m - \omega')^2 = (\omega - \omega')^2 + 2m(\omega - \omega') + \cancel{m^2}$$

$$= E^2 = p^2 + \cancel{m^2}$$

$$\cancel{k^2} + \cancel{k'^2} - 2kk'\cos\theta = \cancel{\omega^2 + \omega'^2} - 2\omega\omega' + 2m(\omega - \omega')$$

$\div\, 2\omega\omega'$

$$\cancel{-}\cos\theta = \cancel{-}1 + m\left(\frac{1}{\omega} - \frac{1}{\omega'}\right) = 1 - m\left(\frac{1}{\omega'} - \frac{1}{\omega}\right)$$

$$\lambda' - \lambda = \frac{2\pi}{m}(1 - \cos\theta) = \frac{2\pi\hbar}{mc}(1 - \cos\theta) = \frac{h}{mc}(1 - \cos\theta)$$