9/7/2016

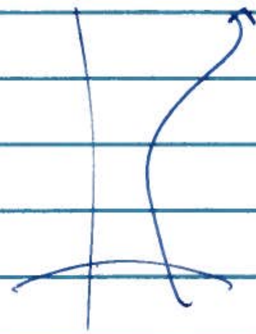
$V = V^\mu$. 4-vector

$\hat{V} = V^\mu \hat{e}_\mu$

$\hat{V}$ = (abstract) vector

V^μ = components

$\hat{e}_\mu$ = basis vector.



$x^\mu(\lambda)$ parametrized curve.

$V^\mu = \frac{dx^\mu}{d\lambda}$ tangent <u>vector</u>.

coordinate transformation

$x'^\nu = x'^\nu(x^\mu)$ $x'^\nu = \Lambda^\nu_\mu x^\mu$

$\frac{dx'^\nu}{d\lambda} = \frac{\partial x'^\nu}{\partial x^\mu} \frac{dx^\mu}{d\lambda}$ $V'^\nu = \Lambda^\nu_\mu V^\mu$

$\hat{V} = \frac{d\hat{x}}{d\lambda}$ is <u>geometric</u> thing $\underline{V^\mu}$ $\underline{V'^\nu}$ are just <u>components</u> in different coords.

$\hat{V} = V^\mu \hat{e}_\mu = V'^\nu \hat{e}'_\nu = \Lambda^\nu_\mu V^\mu \hat{e}'_\nu$

$\Rightarrow \hat{e}_\mu = \Lambda^{\nu'}_\mu \hat{e}'_{\nu'} \rightarrow \hat{e}'_{\nu'} = (\Lambda^{-1})^\mu_{\nu'} \hat{e}_\mu$

V^μ

Index exercises ("calisthenics")

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad \underline{x' = \Lambda x.}$$

$$x'_{\mu} = \eta_{\mu\alpha} x'^{\alpha} = \eta_{\mu\alpha} \Lambda^{\alpha}{}_{\beta} x^{\beta} = \eta_{\mu\alpha} \Lambda^{\alpha}{}_{\beta} \eta^{\beta\nu} x_{\nu}$$

$$\boxed{x'_{\mu} = \Lambda_{\mu}{}^{\nu} x_{\nu}}$$

$$x' \cdot y' = x'_{\mu} y'^{\mu} = (\Lambda_{\mu}{}^{\alpha} x_{\alpha})(\Lambda^{\mu}{}_{\beta} y^{\beta})$$

$$= \Lambda_{\mu}{}^{\alpha} \Lambda^{\mu}{}_{\beta} \cdot x_{\alpha} y^{\beta} = \left((\Lambda^{T})^{\alpha}{}_{\mu} \Lambda^{\mu}{}_{\beta}\right) \underline{x_{\alpha} y^{\beta}}$$

$$\boxed{\Lambda_{\mu}{}^{\alpha} \Lambda^{\mu}{}_{\beta} = \delta^{\alpha}{}_{\beta}} \qquad \Lambda^{T}\Lambda = 1$$

$$\Lambda^{-1} = \Lambda^{T}$$

$$(\Lambda^{-1})^{\mu}{}_{\nu} = (\Lambda^{T})^{\mu}{}_{\nu} = \Lambda_{\nu}{}^{\mu} = \underline{\eta_{\nu\alpha} \Lambda^{\alpha}{}_{\beta} \eta^{\beta\mu}}.$$

x'_{μ}, y^{μ} transform inversely.

v^{μ}, $\hat{e}_{\mu}$ transform inversely.

$$\nabla_{\mu} f = \frac{\partial f}{\partial x^{\mu}} = f_{,\mu} \qquad \boxed{\nabla' f = \frac{\partial f}{\partial x'^{\mu}} = \frac{\partial f}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x'^{\mu}} = (\Lambda^{-1})(\nabla f)}$$

$\hat{v} \in V$ vector space.

V^*: dual space. $\tilde{\omega} \in V^*$ $\boxed{\tilde{\omega}(\hat{v}) \in \mathbb{R}}$

$(\tilde{\omega} : V \to \mathbb{R})$.

linear map. $\tilde{\omega}(a\hat{u} + b\hat{v}) = a\,\tilde{\omega}(\hat{u}) + b\,\tilde{\omega}(\hat{v})$

linear $(a\tilde{\omega} + b\tilde{\eta})\hat{v} = a\,\tilde{\omega}(\hat{v}) + b\,\tilde{\eta}(\hat{v})$

Basis $\tilde{\omega}^\mu$ $\tilde{\eta} = \eta_\mu \tilde{\omega}^\mu$ (Carroll. $\tilde{\theta}^\mu$)

$\boxed{\tilde{\omega}^\mu(\hat{e}_\nu) = \delta^\mu{}_\nu}$ require. (choose)

$\tilde{\eta}(\hat{e}_\nu) = (\eta_\mu \tilde{\omega}^\mu)(\hat{e}_\nu) = \eta_\mu \delta^\mu{}_\nu = \eta_\nu$

$\tilde{\eta}(\hat{v}) = (\eta_\mu \tilde{\omega}^\mu)(v^\nu \hat{e}_\nu) = \eta_\mu v^\nu \tilde{\omega}^\mu(\hat{e}_\nu)$

$= \eta_\mu v^\nu \delta^\mu{}_\nu = \eta_\mu v^\mu.$

V maps $V^* \to \mathbb{R}$ $\hat{v}(\tilde{\omega}) \equiv \tilde{\omega}(\hat{v}) = \eta_\mu v^\mu$

$V^{**} = V$. (finite dimensional)

Tensor

$$T'^{\alpha\beta\cdots}{}_{\gamma\delta\cdots} = \Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}\cdots\Lambda_{\gamma}{}^{\rho}\Lambda_{\delta}{}^{\sigma}\cdots T^{\mu\nu\cdots}{}_{\rho\sigma\cdots}$$

(each index transforms as expected)

Trace $\underbrace{R^{\mu}{}_{\alpha\nu\beta}}_{\text{given.}}$ $R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$

$$R'^{\mu}{}_{\alpha\nu\beta} = \Lambda^{\mu}{}_{\rho}\Lambda_{\alpha}{}^{\gamma}\Lambda_{\nu}{}^{\sigma}\Lambda_{\beta}{}^{\delta}\,R^{\rho}{}_{\gamma\sigma\delta}$$

$$R'^{\mu}{}_{\alpha\mu\beta} = \underbrace{\Lambda^{\mu}{}_{\rho}\Lambda_{\alpha}{}^{\gamma}\Lambda_{\mu}{}^{\sigma}}_{\delta^{\sigma}_{\rho}}\Lambda_{\beta}{}^{\delta}\,R^{\rho}{}_{\gamma\sigma\delta}$$

$$\boxed{\underline{R'_{\alpha\beta}} = \Lambda_{\alpha}{}^{\gamma}\Lambda_{\beta}{}^{\delta}R^{\rho}{}_{\gamma\rho\delta} = \underline{\Lambda_{\alpha}{}^{\gamma}\Lambda_{\beta}{}^{\delta}R_{\gamma\delta}}}$$

Trace, then transform or transform, then trace

$\longrightarrow$ Same.

Symmetric. $T_{(\alpha\beta\gamma)} = \frac{1}{n!}\sum_{\text{permutations}}$

$$= \frac{1}{6}\left(T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} + T_{\gamma\alpha\beta} + T_{\alpha\gamma\beta} + T_{\beta\alpha\gamma} + T_{\gamma\beta\alpha}\right)$$

$$T_{[\alpha\beta\gamma]} = \frac{1}{n!}\sum_{\text{permutations}}(-1)^{P}$$

$$= \frac{1}{6}(+\ +\ +\ -\ -\ -)$$