9/12/2010 Fields $\phi, \psi, A_\mu, g_{\mu\nu}$. $(\vec{x})$.

$$S = \int dt\, L(q, \dot{q}) \rightarrow S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi).$$

$$\sum_i \frac{1}{2} \dot{q}_i^2 \rightarrow \int d^3x\, \frac{1}{2} \dot{\phi}(\vec{x})^2$$

$\vec{x}$ = label. (not dynamical variable).

Lorentz invariant (covariant) Theory.

$\Longleftrightarrow$ $\mathcal{L}$: Lorentz scalar $\rightarrow \dot{\phi}^2 - |\vec{\nabla}\phi|^2$

$$\sum_i (q_i - q_j)^2 \rightarrow \int d^3x\, |\vec{\nabla}\phi|^2 \quad \text{"strain"}$$

$$\boxed{\mathcal{L} = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)}$$

signature (always $+\dot{\phi}^2$)

$$S[\phi] = \int d^4x \left(\frac{1}{2}\eta^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi - V(\phi) \right]$$

$$S[\phi + \delta\phi] - S[\phi] = \delta S$$

$$= \int d^4x \left[-\frac{1}{2}\eta^{\alpha\beta}\partial_\alpha(\delta\phi)\partial_\beta\phi - \frac{1}{2}\eta^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta(\delta\phi) - \frac{\partial V}{\delta\phi}\cdot\delta\phi \right)$$

$\partial_\alpha(\delta\phi) = \delta(\partial_\alpha\phi)$

$$\delta S = \int d^4x \left[- \partial_\alpha (\delta\phi)\, \partial^\alpha \phi - \frac{\partial V}{\partial \phi} \cdot \delta\phi \right)$$

parts.

$$\delta S = \int d^4x \left[+ \partial_\alpha (\partial^\alpha \phi - \frac{\partial V}{\delta \phi} \right] \cdot \delta\phi = 0 .$$

$$- \oint_{\Sigma :\ \text{surface}} d^3\Sigma_\alpha \cdot (\partial^\alpha \phi \cdot \delta\phi)$$

$\delta\phi$ arbitrary. $\rightarrow$ $[\ldots] = 0$

$$\partial^\alpha \partial_\alpha \phi = \Box^2 \phi = \frac{\partial V}{\delta\phi} , \qquad \boxed{\ddot{\phi} - \nabla^2 \phi = - \frac{\partial V}{\delta \phi}}$$

$-\partial_t^2 + \nabla^2$

$V = \frac{1}{2} m^2 \phi^2$ (harmonic oscillator).

$$\boxed{\ddot{\phi} - \nabla^2 \phi = - m^2 \phi} \quad \text{Klein-Gordon.}$$

$e^{+i(\vec{k}\cdot\vec{x} - \omega t)} = e^{i k_\mu x^\mu}$ $\rightarrow$ $\boxed{-\omega^2 + k^2 = m^2}$

$$\boxed{\omega^2 = k^2 + m^2}$$

$$\boxed{E^2 = p^2 + m^2}$$

$\delta\phi$ Surface: initial value problem. $\phi = \phi_0(\vec{x})$ on Σ_0

$\rightarrow$ $\delta\phi = 0$ on Σ_0

3

Functional derivative.

$\frac{\partial q_i}{\partial q_j} = \delta_{ij}$ $\qquad\qquad$ $\frac{\delta\phi(x)}{\delta\phi(x')} = \delta^{(4)}(x-x')$

$$\delta S = \int d^4x' \, \frac{\delta S}{\delta\phi(x')} \, \delta\phi(x')$$

$$= \int d^4x' \left[\frac{\partial \mathcal{L}}{\partial\phi} - \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \right) \right] \delta\phi(x')$$

$$\boxed{\frac{\delta S}{\delta\phi(x)} = \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \right)}$$

$$\lim_{\varepsilon\to 0} \frac{S[\phi+\varepsilon\delta\phi] - S[\phi]}{\varepsilon} = \int d^4x \, \frac{\delta F}{\delta\phi} \, \delta\phi(x)$$

Maxwell. $\quad$ MKS. $\quad \frac{1}{4\pi\varepsilon_0} \quad \frac{\mu_0}{4\pi}$.

Gaussian $\quad 4\pi\varepsilon_0 = 1 \quad \frac{\mu_0}{4\pi} = 1$.

Lorentz. $\quad \varepsilon_0 = 1. \quad \mu_0 = 1. \quad \leftarrow$ QFT. Carroll.

Gaussian. $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu$$

$$\partial_\mu F^{\nu\mu} = (4\pi) J^\nu.$$

$$\mathcal{L} = -\frac{1}{4[4\pi]}\eta^{\alpha\rho}\eta^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + A_\mu J^\mu$$

$$\cancel{\frac{\partial \mathcal{L}}{\partial A_\mu} = J^\mu} \qquad \boxed{\frac{\partial \mathcal{L}}{\partial A_\nu} = J^\nu}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{16\pi}\eta^{\alpha\rho}\eta^{\beta\sigma}\cdot 2F_{\alpha\beta}\frac{\partial F_{\rho\sigma}}{\partial(\partial_\mu A_\nu)}$$

$$= -\frac{1}{16\pi}\eta^{\alpha\rho}\eta^{\beta\sigma}\cdot 2F_{\alpha\beta}\left[\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho\right]$$

$$= -\frac{1}{8\pi}F^{\sigma\rho}\left(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho\right) = -\frac{1}{4\pi}F^{\mu\nu}$$

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}\right) - \frac{\partial \mathcal{L}}{\partial A_\nu} = -\frac{1}{4\pi}\partial_\mu F^{\mu\nu} - J^\nu = 0$$

$$\boxed{\partial_\mu F^{\nu\mu} = [4\pi]J^\nu}$$

J.D.J. (11.141) $\partial_\alpha F^{\alpha\beta} = \underline{4\pi} J^\beta$.

Signature

units

$$p = \frac{\partial L}{\partial \dot q} = \dot q \qquad\qquad \pi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \dot\phi$$

$$H = p\dot q - L \qquad\qquad \mathcal{H} = \pi\dot\phi - \mathcal{L}$$

$$\mathcal{H} = \dot\phi^2 - \left(\tfrac{1}{2}\dot\phi^2 - \tfrac{1}{2}|\vec\nabla\phi|^2 - V\right) = \tfrac{1}{2}\dot\phi^2 + \tfrac{1}{2}|\vec\nabla\phi|^2 + V$$

$$\boxed{u = \tfrac{1}{2}\dot\phi^2 + \tfrac{1}{2}|\vec\nabla\phi|^2 + V}$$

energy density   kinetic   shear   potential

$$\frac{\partial u}{\partial t} = \dot\phi\ddot\phi + \vec\nabla\phi\cdot\vec\nabla\dot\phi + \dot\phi\frac{\partial V}{\partial\phi}$$

$$\ddot\phi - \nabla^2\phi = -\frac{\partial V}{\partial\phi} \Rightarrow \dot\phi\left(\ddot\phi + \frac{\partial V}{\partial\phi}\right) = \underline{\dot\phi\nabla^2\phi}$$

$$\frac{\partial u}{\partial t} = \dot\phi\nabla^2\phi + \vec\nabla\phi\cdot\vec\nabla\dot\phi = \vec\nabla\cdot(\dot\phi\vec\nabla\phi)$$

$$\underline{\vec S = -\dot\phi\vec\nabla\phi} \qquad \boxed{\frac{\partial u}{\partial t} + \vec\nabla\cdot\vec S = 0}$$

u is one component of a spacetime tensor (00).

$$\boxed{T^{\mu\nu} = -(\partial^\mu\phi)\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)} + \eta^{\mu\nu}\mathcal{L}}$$

$$T^{00} = -(-\dot\phi)(\dot\phi) + (-1)\left(\tfrac{1}{2}\dot\phi^2 - \tfrac{1}{2}|\vec\nabla\phi|^2 - V\right)$$

$$= \tfrac{1}{2}\dot\phi^2 + \tfrac{1}{2}|\vec\nabla\phi|^2 + V \qquad = \underline{u = \varepsilon}$$

$$T^{0i} = -(-\dot\phi)(-\nabla^i\phi) = -\dot\phi\,\nabla^i\phi = \underline{S^i}$$

$$T^{i0} = -(\nabla^i\phi)(\dot\phi) = -\dot\phi\,\nabla^i\phi = \underline{g^i}$$

stress-energy tensor   energy-momentum tensor

Each μ:

$$\partial_\nu T^{\mu\nu} = T^{\mu\nu}{}_{,\nu} = \partial_\nu\left[-\partial^\mu\phi\,\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)} + \eta^{\mu\nu}\mathcal{L}\right]$$

$$= -\partial_\nu(\partial^\mu\phi)\cdot\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)} - (\partial^\mu\phi)\,\partial_\nu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}\right)$$

$$+ \eta^{\mu\nu}\left(\frac{\partial\mathcal{L}}{\partial\phi}\partial_\nu\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\rho\phi)}\partial_\nu(\partial_\rho\phi)\right)$$

$$= -\cancel{\phi^{,\mu}{}_{,\nu}\cdot\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)}} - \cancel{(\partial^\mu\phi)\frac{\partial\mathcal{L}}{\partial\phi}} + \cancel{(\partial^\mu\phi)\frac{\partial\mathcal{L}}{\partial\phi}} + \cancel{\frac{\partial\mathcal{L}}{\partial(\partial_\rho\phi)}\partial^\mu\partial_\rho\phi} = 0.$$

$T^{\mu\nu}$ = density (flux) of 4-momentum p^μ across a surface of constant x^ν

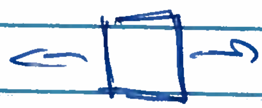
T^{00} energy per (x-y-z) energy/volume

T^{0i} flow of energy per time per (area $\perp x^i$)

T^{i0} density of p^i per volume.

T^{ij} rate of flow of p^i per area $\perp x^j$)

= (force)^i per area $\perp x^j$

$T_{xx} \Leftrightarrow$ pressure $T_{xy} \Leftrightarrow$ shear

Isotropic perfect fluid at rest -

$T^{00} = \rho \qquad T^{0i} = T^{i0} = 0 \qquad T^{ij} = p \cdot \delta^{ij}$

$$T = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \qquad T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$

$$= \underbrace{\rho\, u^\mu u^\nu}_{P_\parallel} + \underbrace{p\,(\eta^{\mu\nu} + u^\mu u^\nu)}_{P_\perp}.$$