1/14/2016

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$\delta S = 0 \rightarrow \dfrac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\dfrac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0 \quad = \dfrac{\delta S}{\delta \phi(x)}$

$$\boxed{T^{\mu\nu} = -\partial^\mu \phi \cdot \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} + \eta^{\mu\nu} \mathcal{L}}$$

no explicit x_μ

$\mathcal{L}(t+c) = \mathcal{L}(t)$

$\mathcal{L}(\vec{x}+\vec{c}) = \mathcal{L}(\vec{x})$

$\partial_\mu \mathcal{L} = \dfrac{\partial \mathcal{L}}{\partial \phi} \partial_\mu \phi + \dfrac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \partial_\mu (\partial_\nu \phi) \Leftrightarrow \partial_\nu T^{\mu\nu} = 0$

$= \partial_\nu \left(\dfrac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \right) \partial_\mu \phi + \dfrac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \partial_\nu (\partial_\mu \phi)$

$= \partial_\nu \left[\partial_\mu \phi \cdot \dfrac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \right]$.

$\boxed{\text{symmetry} \Leftrightarrow \text{conservation}}$

$T^{\mu\nu}$ = density (flux) of 4-momentum p^μ across a surface of constant x^ν.

T^{00}. Energy · per (x-y-z) energy per volume

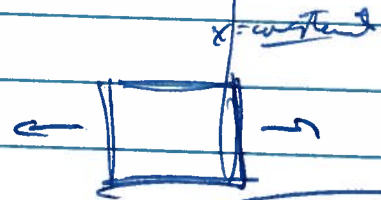
T^{0i} Flow of energy per time per area $\perp x^i$

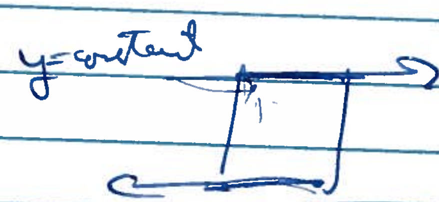
~~$d\Sigma^i = \epsilon \ldots$~~

T^{i0} = density of p^i per volume.

T^{ij}: rate of flow of p^i per time per area $\perp x^j$

= $(\text{Force})^i$ per area $\perp x^j$.





T_{xx} = pressure T_{xy} = shear

Isotropic fluid. $T^{0i} = T^{i0} = 0$ at rest

$T^{00} = \rho \qquad T^{ij} = p \cdot \delta^{ij} \qquad T^{\mu\nu} u_\mu u_\nu = \rho$

$$T = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \qquad T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$

$$T^{\mu\nu} = \rho\, u^\mu u^\nu + p\left(\eta^{\mu\nu} + u^\mu u^\nu\right)$$

$$\underbrace{\begin{pmatrix} 1 & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}}_{P_\parallel} \qquad \underbrace{\begin{pmatrix} & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}}_{P_\perp}$$

Scalar <u>field</u>. $\phi = c \cdot e^{ik\cdot x}$

$$T^{\mu\nu} = \phi^{,\mu}\phi^{,\nu} + \eta^{\mu\nu}\mathcal{L} \qquad \omega,\ \omega u^{\alpha}$$

$$= |c|^2 \left(k^\mu k^\nu + \eta^{\mu\nu}\left(\tfrac{1}{2}\omega^2 - \tfrac{1}{2}|\vec{k}|^2\right)\right)$$

$$\langle T^{\mu\nu}\rangle = \langle |c|^2\rangle \cdot \omega^2 \begin{pmatrix} 1 & & & \\ & \tfrac{1}{3} & & \\ & & \tfrac{1}{3} & \\ & & & \tfrac{1}{3} \end{pmatrix}$$

$\langle \hat{u}^i\rangle = 0$ $\quad \langle \hat{u}^i \hat{u}^j\rangle = \frac{1}{3}\delta^{ij}$ $\quad p = \frac{1}{3}\rho$

<u>radiation gas</u>.

$$T^\mu{}_\mu = (\rho + p)\underbrace{u^\mu u_\mu}_{-1} + p \cdot \underbrace{\eta^\mu{}_\mu}_{\delta^\mu_\mu = 4} = \underline{-\rho + 3p}$$

$$\boxed{T = -(\rho - 3p)} \qquad p = \tfrac{1}{3}\rho \quad T = 0$$

$$p < \tfrac{1}{3}\rho \quad T < 0$$

<u>Strong energy condition</u>

$\rho \geq 0$ $\quad$ weak energy condition

$p < \rho$

④

Branches. $\left\{ \begin{array}{l} \text{more field stuff} \\ \text{more fluid stuff} \end{array} \right.$

$$T^{\mu\nu} = (\rho+p)u^\mu u^\nu + p\,\eta^{\mu\nu}$$

u^μ: comoving with fluid

$$T^{\mu\nu}{}_{,\nu} = \partial_\nu\left[(\rho+p)u^\mu u^\nu + p\,\eta^{\mu\nu}\right]$$

$$= u^\mu u^\nu \partial_\nu(\rho+p) + (\rho+p)u^\nu \partial_\nu u^\mu + (\rho+p)u^\mu(\partial_\nu u^\nu) + \partial^\mu p .$$

Project on u^μ

$$\underline{u_\mu T^{\mu\nu}{}_{,\nu}} = (-1)u^\nu \partial_\nu(\rho+p) + (\rho+p)u^\nu \cancel{(u_\mu \partial_\nu u^\mu)} + (\rho+p)(-1)(\partial_\nu u^\nu) + u_\mu \partial^\mu p$$

$$u^\mu \partial_\mu f = \underline{\frac{\partial f}{\partial x^\mu}\cdot\frac{dx^\mu}{d\tau}} = \nabla_u f = \frac{df}{d\tau}$$

$$-\frac{d\rho}{d\tau} - \cancel{\frac{dp}{d\tau}} + (\rho+p)(-1)\,\underline{\nabla\cdot u} + \cancel{\frac{dp}{d\tau}} = 0$$

$$\boxed{\frac{d\rho}{d\tau} + (\rho+p)\,\nabla\cdot u = 0} \qquad (1.118)$$

⑤

Project $\perp u^\mu$. $\cancel{\delta^\mu_\nu + u^\mu u_\nu}$ $\quad \boxed{(\delta^\lambda_\mu + u^\lambda u_\mu)} u^\mu = u^\lambda + \cancel{u^\lambda (u^\mu u_\mu)}$

$$\cancel{(\delta^\lambda_\mu + u^\lambda u_\mu) \left(u^\mu u^\nu \partial_\nu (\rho + p) \right.}$$

$$\cancel{(\delta^\lambda_\mu + u^\lambda u_\mu)} (\rho + p) u^\nu \partial_\nu u^\mu \Big) = 0$$

$$+ \cancel{(\delta^\lambda_\mu + u^\lambda u_\mu)(\rho + p) u^\mu \partial_\nu u^\nu} + (\delta^\lambda_\mu + u^\lambda u_\mu) \partial^\mu p$$

$$\boxed{(\rho + p) \frac{du^\lambda}{d\tau} + \partial^\lambda p + u^\lambda \frac{dp}{d\tau} = 0.} \quad (1.123)$$

$$u^\mu \partial_\mu \rho = \gamma \cdot \frac{\partial \rho}{\partial t} + \gamma \vec{v} \cdot \vec{\nabla} v \simeq \frac{\partial \rho}{\partial t} + (\vec{v} \cdot \vec{\nabla}) v$$

$v \ll c$

$$\partial \cdot u = \frac{\partial \gamma}{\partial t} + \vec{\nabla} \cdot (\gamma \vec{v}) \simeq \vec{\nabla} \cdot \vec{v}$$

$$\hookrightarrow \left(\tfrac{1}{2}\right)(1 - v^2)^{3/2} \left(2 \vec{v} \cdot \frac{d\vec{v}}{dt}\right) = \gamma^3 \vec{v} \cdot \frac{d\vec{v}}{dt}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0}$$

continuity. $\vec{J} = \rho \vec{v}$

$p \ll \rho$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) + \vec{v} \cancel{\left(\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho \right)} + \vec{\nabla} p = 0$$

$$\boxed{\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = -\vec{\nabla} p}$$

Euler

conserved number: (n) in local rest frame
$\Leftarrow$ conserved.

$N^\mu = n u^\mu$ $\qquad n = -u \cdot N$ $\qquad$ invariant.

$\partial_\mu N^\mu = u^\mu \partial_\mu n + n \partial_\mu u^\mu = 0$ (conserved)

$$\Rightarrow \boxed{\partial_\mu u^\mu = \nabla \cdot u = -\frac{1}{n} \frac{dn}{d\tau}}$$

$$\frac{d\rho}{d\tau} + (\rho + p)\left(-\frac{1}{n}\frac{dn}{d\tau}\right) = 0$$

$$\boxed{\frac{d\rho}{d\tau} = \frac{\rho + p}{n} \frac{dn}{d\tau}}$$

$$d\left(\frac{\rho}{n}\right) = \frac{d\rho}{n} - \rho \frac{dn}{n^2}$$

$$= \left(\frac{\cancel{\rho} + p}{n^2}\right) dn - \cancel{\frac{\rho}{n^2} dn} = -p \cdot d\left(\frac{1}{n}\right)$$

$$\boxed{d\left(\frac{\rho}{n}\right) = -p \, d\left(\frac{1}{n}\right)} + T \, d\left(\frac{s}{n}\right)$$

$dE = -p \, dV$

ideal fluid. $\frac{s}{n}$ = constant