9/16/2016

$x^\mu(\tau)$

$u^\mu = \frac{dx^\mu}{d\tau}$

J.D.J (12.139) $\quad J^\mu(x) = ec\int d\tau \, u^\mu(\tau)\, \delta^{(4)}(x^\mu - r^\mu(\tau))$

$$\longrightarrow T^{\mu\nu} = m \int d\tau \, u^\mu(\tau)\, u^\nu(\tau)\, \delta^{(4)}(x^\mu - r^\mu(\tau))$$

$$\int d\tau \, \delta(t - r^0(\tau)) = \int d\tau \frac{\delta(\tau - \tau_0)}{\left|\frac{dr^0}{d\tau}\right|} = \frac{1}{u^0} = \frac{1}{\gamma}.$$

$$T^{\mu\nu} = \sum_a \frac{m\, u_a^\mu u_a^\nu}{u_a^0} \delta^{(3)}(\vec{x} - \vec{r}_a(t)) = \sum_a \frac{p_a^\mu p_a^\nu}{p_a^0} \delta^{(3)}(\vec{x} - \vec{r}_a(t))$$

(Carroll. (1.173). Problem 1.9)

$$T^{00} = \sum_a \gamma_a m_a \, \delta^{(3)}(\vec{x} - \vec{r}_a(t)) \rightarrow \langle \gamma^2 m n \rangle = \rho$$

$$T^{0i} = T^{i0} = \sum_a \gamma_a m_a \vec{v}_a \, \delta^{(3)}(\vec{x} - \vec{r}_a(t)) \rightarrow \langle \gamma^2 m n \vec{v} \rangle = \rho \langle v^i \rangle = \rho \bar{v}^i$$

$$T^{ij} = \sum_a \gamma m_a v_a^i v_a^j \, \delta^{(3)}(\vec{x} - \vec{r}_a) \rightarrow \langle \gamma^2 m n v^i v^j \rangle$$

$$= \gamma m n \bar{v}^i \bar{v}^j + \langle \gamma m n \, \delta v^i \delta v^j \rangle = \rho \bar{v}^i \bar{v}^j + p\, \delta^{ij}$$

$$T = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

$\vec{v}$ = constant

$\rho u^\mu u^\nu$

$\vec{v} = 0$ $\quad \begin{pmatrix} \rho & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$ $\quad$ dust

$$T^{\mu\nu} = \rho u^\mu u^\nu + p(u^\mu u^\nu + \eta^{\mu\nu}) = [\,]_{\gamma_{\gamma_\rho}}$$

$$T^{\mu\nu}{}_{,\nu} = 0 \quad \rightarrow$$

$$\boxed{\frac{d\rho}{d\tau} + (\rho + p)\nabla\cdot u = 0} \qquad (1.118)$$

$$\boxed{(\rho+p)\frac{du^\mu}{d\tau} = -\partial^\mu p - u^\mu \frac{\partial p}{\partial \tau}} \qquad (1.123)$$

$$\boxed{-(u^\mu u^\nu + \eta^{\mu\nu})\partial_\nu p}$$

enforces $\boxed{u_\mu \frac{du^\mu}{d\tau} = 0}$?

$\nabla\cdot u \iff$ conserved number.

$\boxed{N^\mu = n u^\mu}$ number current. $\boxed{n = -u\cdot N}$

$$\partial_\mu N^\mu = 0 = u^\mu \partial_\mu n + n\,\partial_\mu u^\mu = \frac{dn}{d\tau} + n\,\nabla\cdot u$$

$$\Rightarrow \nabla\cdot u = \partial_\mu u^\mu = -\frac{1}{n}\frac{dn}{d\tau}$$

$$\frac{d\rho}{d\tau} + (\rho+p)\left(-\frac{1}{n}\frac{dn}{d\tau}\right) = 0$$

$$\boxed{\frac{d\rho}{d\tau} = \frac{\rho+p}{n}\frac{dn}{d\tau}}$$

③

$$d\left(\frac{\rho}{n}\right) = \frac{d\rho}{n} - \rho\frac{dn}{n^2}$$

$$= \left(\frac{\cancel{\rho}+p}{n^2}\right)dn - \cancel{\rho\frac{dn}{n^2}} = -p\, d\left(\frac{1}{n}\right)$$

$$d\left(\frac{\rho}{n}\right) = -p\, d\left(\frac{1}{n}\right) + T\,d\left(\frac{s}{n}\right).$$

perfect fluid. (adiabatic)

ideal fluid $\frac{s}{n}$ = constant

Thermo. pV^{γ} = constant

$$\boxed{p \propto \rho^{\gamma}}$$

$p = nkT$

Relativistic. $\gamma_1, \gamma_2, \gamma_3$

$$\boxed{\frac{\partial \ln p}{\partial \ln n} = \Gamma_1}$$

$\rho \propto T^4$

$n \propto T^3$

$$\frac{\partial \ln p}{\partial \ln T} = \frac{\Gamma_2}{\Gamma_2 - 1}$$

$$\frac{\partial \ln T}{\partial \ln n} = (\Gamma_3 - 1)$$

4

EQUs: $\mathcal{L} = -\frac{1}{16\pi} \eta^{\alpha\rho}\eta^{\beta\sigma} F_{\alpha\beta} F_{\rho\sigma} + A_\alpha J^\alpha$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha .$$

$\pi^\alpha = \frac{\partial \mathcal{L}}{\partial \dot{A}_\alpha}$ $\quad \pi^0 = 0 \quad$ constrained system.

$\rho=0 \ \sigma=i \qquad \rho=i \ \sigma=0$

$$\pi^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = -\frac{1}{16\pi} \eta^{\alpha\rho}\eta^{\beta\sigma} F_{\alpha\beta} \left(\delta^0_\rho \delta^i_\sigma - \delta^i_\rho \delta^0_\sigma \right) \times 2$$

$$= -\frac{1}{16\pi} \eta^{\alpha 0} \cdot \eta^{\beta i} F_{\alpha\beta} \times 4$$

$$= -\frac{1}{4\pi} F^{0i} = -\frac{1}{4\pi} \left(\partial^0 A^i - \partial^i A^0 \right)$$

$$= -\frac{1}{4\pi} \left(-\frac{\partial A^i}{\partial t} - \nabla^i \Phi \right) = -\frac{1}{4\pi} E^i$$

$$\mathcal{H} = \pi^\mu \dot{A}_\mu - \mathcal{L} .$$

$$T^{\mu\nu} = -(\partial^\nu A_\rho) \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} + \eta^{\mu\nu} \mathcal{L} .$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4\pi} F^{\mu\nu} \qquad \left(\begin{matrix} 0 \to \mu \\ i \to \nu \end{matrix} \right)$$

(5)

$$T^{\mu\nu} = -(\partial^\mu A_\rho)\left(-\frac{1}{4\pi}F^{\nu\rho}\right) + \eta^{\mu\nu}\mathcal{L}$$

$$T^{\mu\nu} = \frac{1}{4\pi}\partial^\mu A^\rho F^\nu{}_\rho - \frac{1}{16\pi}\eta^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

canonical

not gauge invariant
not symmetric (need)

spin density.
$$\boxed{J_{\alpha\beta}{}^\mu = x_\alpha T_\beta{}^\mu - x_\beta T_\alpha{}^\mu + S_{\alpha\beta}{}^\mu}$$

$$J_{\alpha\beta}{}^\mu{}_{,\mu} = T_{\beta\alpha} - T_{\alpha\beta} + \partial_\mu S_{\alpha\beta}{}^\mu = 0$$

$$G_{\mu\nu\rho} = \frac{1}{2}(S_{\nu\rho\mu} + S_{\mu\rho\nu} - S_{\mu\nu\rho})$$

$$\Theta^{\mu\nu} = T^{\mu\nu} + \partial_\rho G^{\mu\nu\rho}$$

$\exp(\alpha M)A$

$$S_{\alpha\beta}{}^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu A^\lambda)}[M_{\alpha\beta}]^\lambda{}_\sigma A^\sigma = F^\mu{}_\alpha A_\beta - F^\mu{}_\beta A_\alpha$$

$\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\lambda)}\frac{dA_\lambda}{d\alpha} \to \alpha M$

$$\hookrightarrow\left(-\eta^\lambda_\alpha\eta_{\beta\sigma} + \eta^\lambda_\beta\eta_{\alpha\sigma}\right)$$

$$\partial_\nu\Theta^{\mu\nu} = \partial_\nu T^{\mu\nu} + \partial_\nu\partial_\rho G^{\mu\nu\rho} = \partial_\nu T^{\mu\nu} = 0$$

$$\Theta^{\mu\nu} - \Theta^{\nu\mu} = T^{\mu\nu} - T^{\nu\mu} - \partial_\rho S^{\mu\nu\rho} = 0.$$