

9/23/2016

$$u^\mu = \frac{dx^\mu}{dt} = \left(\frac{dx^0}{dt}, \frac{dx^i}{dt} \right)$$

$$\partial_\mu f = \frac{\partial f}{\partial x^\mu} = \frac{\partial f}{\partial x^i} \frac{\partial x^i}{\partial x^\mu}$$

In general, don't preserve η .

$$g_{\alpha\beta} = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} g_{\mu\nu}$$

$$e_\alpha^\mu = \frac{\partial x^\mu}{\partial x'^\alpha} = \frac{\partial x^\mu}{\partial x^\nu} e_\alpha^\nu$$

$$g^{\alpha\beta} = g_{\mu\nu} \delta x^\mu \otimes \delta x^\nu$$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

$x = r \cos \phi$
 $y = r \sin \phi \rightarrow g = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$

$$\hat{V} \cdot \hat{V} = g_{\alpha\beta} v^\alpha v^\beta$$

coordinate basis.
 v^α, v^β with even same units.

$$\partial_\mu f = \begin{pmatrix} \frac{\partial f}{\partial x^0} \\ \frac{\partial f}{\partial x^1} \\ \frac{\partial f}{\partial x^2} \\ \frac{\partial f}{\partial x^3} \end{pmatrix}$$

with same units.

$$\partial_\mu \partial^i f = g^{ij} \partial_j f \rightarrow$$

$$\begin{pmatrix} \frac{\partial f}{\partial x^0} \\ \frac{\partial f}{\partial x^1} \\ \frac{\partial f}{\partial x^2} \\ \frac{\partial f}{\partial x^3} \end{pmatrix}$$

$$\partial_\mu f = \frac{\partial f}{\partial x^\mu} \quad \partial^\mu f = (\partial^\alpha f \cdot \partial_\alpha f)^{1/2} = \left(\frac{\partial f}{\partial x^\alpha} \frac{\partial f}{\partial x^\alpha} \right)^{1/2}$$

2.

New trouble. derivatives.

$$\hat{V} = V^\alpha \hat{e}_\alpha \quad \frac{\partial \hat{V}}{\partial x^\beta} = \frac{\partial V^\alpha}{\partial x^\beta} \hat{e}_\alpha + V^\alpha \frac{\partial \hat{e}_\alpha}{\partial x^\beta}$$

$$\frac{\partial \hat{e}_\alpha}{\partial x^\beta} = \Gamma_{\beta\alpha}^\mu \hat{e}_\mu \quad \left(\begin{array}{l} \text{vector-like object} \\ \rightarrow \text{linear combination of } \hat{e}^\mu \end{array} \right)$$

$$\frac{\partial \hat{V}}{\partial x^\beta} = \left(V^\mu_{;\beta} + \Gamma_{\beta\alpha}^\mu V^\alpha \right) \hat{e}_\mu$$

$$\nabla_\beta V^\mu = V^\mu_{;\beta} \equiv V^\mu_{;\beta} + \Gamma_{\beta\alpha}^\mu V^\alpha$$

"Covariant derivative"

$$V^\mu_{;\alpha} \neq \text{tensor} \quad \Gamma_{\beta\alpha}^\mu \neq \text{tensor} \quad V^\mu_{;\alpha} \text{ tensor}$$

$$\nabla_\beta \hat{V} = (1,1) \text{ tensor} \equiv (\nabla \hat{V})^\mu_\beta = V^\mu_{;\beta}$$

$$\begin{aligned} x &= r \cos\phi \\ y &= r \sin\phi \end{aligned}$$

$$\hat{e}_r = \frac{\partial x}{\partial r} \hat{e}_x + \frac{\partial y}{\partial r} \hat{e}_y = \cos\phi \hat{e}_x + \sin\phi \hat{e}_y$$

$$\hat{e}_\phi = \frac{\partial x}{\partial \phi} \hat{e}_x + \frac{\partial y}{\partial \phi} \hat{e}_y = -r \sin\phi \hat{e}_x + r \cos\phi \hat{e}_y$$

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$$\frac{\partial \hat{e}_r}{\partial r} = 0 \quad \frac{\partial \hat{e}_r}{\partial \phi} = -\sin\phi \hat{e}_x + \cos\phi \hat{e}_y = \frac{1}{r} \hat{e}_\phi$$

$$\frac{\partial \hat{e}_\phi}{\partial r} = \frac{1}{r} \hat{e}_\phi \quad \frac{\partial \hat{e}_\phi}{\partial \phi} = -\cos\phi \hat{e}_x - \sin\phi \hat{e}_y = -r \hat{e}_r$$

$$\frac{\partial \hat{e}_i}{\partial x^j} = \Gamma_{\alpha\beta}^{\gamma} \hat{e}_\gamma \quad \frac{\partial \hat{e}_i}{\partial x^j} = \Gamma_{ij}^k \hat{e}_k \quad \left(\Gamma_{rr}^r = \Gamma_{\phi\phi}^r = 0 \right)$$

$\frac{\partial \hat{e}_r}{\partial \phi}$ $\Gamma_{r\phi}^r = 0$ $\Gamma_{\phi\phi}^r = \frac{1}{r}$

$\frac{\partial \hat{e}_\phi}{\partial r}$ $\Gamma_{rr}^\phi = 0$ $\Gamma_{\phi\phi}^\phi = -r$ $\Gamma_{\phi\phi}^r = 0$

$$\nabla_{\hat{e}_i} \hat{e}_j = \nabla_{\mu} \hat{e}_j \quad \nabla_{\hat{e}_i} \hat{e}_j = \nabla_{\hat{e}_i}^2 \hat{e}_j = (\hat{e}_i^k)_{,j} \hat{e}_k$$

$$\hat{e}_r^k{}_{,r} = \hat{e}_r^k{}_{,r} + \cancel{\Gamma_{rr}^r \hat{e}_r^r} + \cancel{\Gamma_{r\phi}^r \hat{e}_\phi^r}$$

$$\hat{e}_\phi^k{}_{,r} = \hat{e}_\phi^k{}_{,r} + \Gamma_{r\phi}^r \hat{e}_r^r + \cancel{\Gamma_{\phi\phi}^r \hat{e}_\phi^r}$$

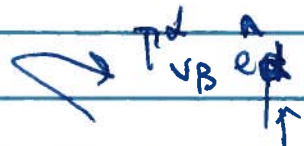
$$\nabla_{\hat{e}_r}^2 \hat{e}_r = (1) \left(\frac{\partial \hat{e}_r}{\partial r} \right) + \left(\frac{1}{r} \right)_{,r} \hat{e}_r + \left(\frac{1}{r} \right) (1) \left(\frac{\partial \hat{e}_r}{\partial r} \right)$$

$$= \frac{\partial^2 \hat{e}_r}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{e}_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \hat{e}_r}{\partial \phi^2}$$

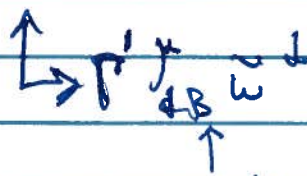
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∇ acts on $\tilde{\omega}$'s.

$$\langle \tilde{\omega}^\mu, \hat{e}_\nu \rangle = g^\mu_\nu$$



$$\frac{\partial}{\partial x^B} \langle \tilde{\omega}^\mu, \hat{e}_\nu \rangle = \left\langle \frac{\partial \tilde{\omega}^\mu}{\partial x^B}, \hat{e}_\nu \right\rangle + \langle \tilde{\omega}^\mu, \frac{\partial \hat{e}_\nu}{\partial x^B} \rangle$$



$$\Gamma^{\mu\alpha}_{\nu B} \delta^\alpha_\nu + \Gamma^{\rho\alpha}_{\nu B} \delta^\alpha_\rho = 0$$

$$\Gamma^{\mu\alpha}_{\nu B} = -\Gamma^{\alpha\mu}_{\nu B}$$

$$\frac{\partial \tilde{\omega}^\mu}{\partial x^B} = -\Gamma^{\alpha\mu}_{\nu B} \tilde{\omega}^\nu$$

~~$\tilde{\omega}^\mu = \omega^\mu$~~

$$\frac{\partial \tilde{\omega}^\mu}{\partial x^B} = \frac{\partial}{\partial x^B} (\omega^\mu) = \left(\frac{\partial \omega^\mu}{\partial x^B} + \omega^\nu \frac{\partial \omega^\mu}{\partial x^A} \right)$$

$$= \left(\frac{\partial \omega^\mu}{\partial x^B} - \Gamma^{\alpha\mu}_{\nu B} \omega^\nu \right) \tilde{\omega}^\nu$$

$$\omega^\mu_{;\nu B} = (\nabla \tilde{\omega}^\mu)_{\nu B} = \omega^\mu_{;\nu B} - \Gamma^{\alpha\mu}_{\nu B} \omega^\alpha$$