10 | 10 | 2016

$$\boxed{\nabla_\alpha \nabla_\beta V^\mu - \nabla_\beta \nabla_\alpha V^\mu = R^\mu{}_{\lambda\alpha\beta} V^\lambda} \quad (3.112)$$

$$\boxed{R^\mu{}_{\lambda\alpha\beta} = -\Gamma^\mu_{\lambda\alpha,\beta} + \Gamma^\mu_{\lambda\beta,\alpha} + \Gamma^\mu_{\nu\alpha}\Gamma^\nu_{\lambda\beta} - \Gamma^\mu_{\nu\beta}\Gamma^\nu_{\lambda\alpha}}$$

(3.143) $T^\lambda_{[\mu\nu]}$

$R_{\mu\lambda\alpha\beta} = R_{[\mu\lambda][\alpha\beta]} = R_{[\alpha\beta][\mu\lambda]}$ 129 130 131

"use your imagination" $R_{\mu[\lambda\alpha\beta]} = 0$

$\frac{1}{12} n^2 (n^2-1)$ $\quad \frac{1}{12} \cdot 16 \cdot 15 = 4 \cdot 5 = 20$

$n=1$ $\quad R_{0000} = 0$

$n=2$ $\quad R_{0101} \quad \frac{1}{12} \cdot 4 \cdot 3 = 1$

$n=3$ $\quad \frac{1}{12} \cdot 9 \cdot 8 = 6$ $\quad R_{0101} \quad R_{0102} \quad R_{0212}$

$R_{0202} \quad R_{0112}$

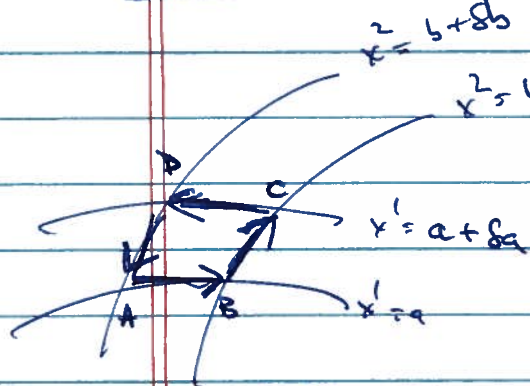
R_{1212}

$$\boxed{\sum_{i=0}^{s-1} C_n^{i+1} C_{s-1}^{i}} = \frac{(n+s-1)!}{(n-1)!\, s!}$$

$$\boxed{\nabla_\alpha \nabla_\beta V^\mu - \nabla_\beta \nabla_\alpha V^\mu = R^\mu{}_{\lambda\alpha\beta} V^\lambda}$$

$\cancel{\partial\partial V}$ $\cancel{\partial V}$.

linear in V.



$$\boxed{\frac{dV^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} V^\mu u^\nu = 0}$$

$$\frac{dx^\nu}{d\lambda} d\lambda = dx^\nu$$

AB: $V^\alpha(B) = V^\alpha(A) + \int_A^B \left(-\Gamma^\alpha_{\mu\nu} V^\mu\right) u^\nu \, d\lambda$

$$= V^\alpha(A) - \int_a^{a+\delta a} \Gamma^\alpha_{\mu 1} V^\mu \cdot dx^1 \Big|_{(x^2=b)}$$

$$\delta V^\alpha_{AB} = -\int_a^{a+\delta a} (dx^1)\, \Gamma^\alpha_{\mu 1} V^\mu \Big|_{(x^2)=b}$$

$$\delta V^\alpha_{BC} = -\int_b^{b+\delta b} (dx^2)\, \Gamma^\alpha_{\mu 2} V^\mu \Big|_{(x^1)=a+\delta a}$$

$$\delta V^\alpha_{CD} = +\int_a^{a+\delta a} (dx^1)\, \Gamma^\alpha_{\mu 1} V^\mu \Big|_{x^2=b+\delta b}$$

$$\delta V^\alpha_{DA} = +\int_b^{b+\delta b} (dx^2)\, \Gamma^\alpha_{\mu 2} V^\mu \Big|_{(x^1)=a}$$

(3)

$$\delta V^\mu = (\delta b)\left[\frac{\partial}{\partial x^\alpha}\left(\Gamma^\mu_{\rho\beta} V^\rho\right) \cdot \delta a\right] - (\delta a)\left[\frac{\partial}{\partial x^\beta}\left(\Gamma^\mu_{\rho\alpha} V^\rho\right) \cdot \delta b\right]$$

$$= \delta a \cdot \delta b \left[\Gamma^\mu_{\rho\beta,\alpha} V^\rho + \Gamma^\mu_{\rho\beta} V^\rho_{,\alpha} - \Gamma^\mu_{\rho\alpha,\beta} \cdot V^\rho - \Gamma^\mu_{\rho\alpha} \cdot V^\rho_{,\beta}\right]$$

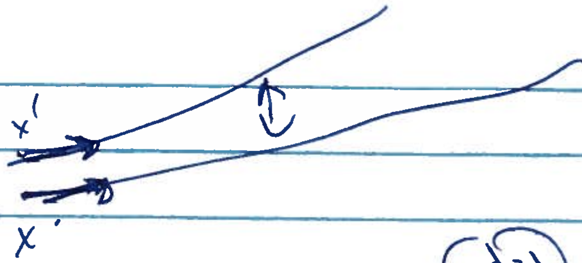
$$= \delta a \cdot \delta b \left[\Gamma^\mu_{\rho\beta,\alpha} V^\rho - \Gamma^\mu_{\rho\beta} \cdot \Gamma^\rho_{\sigma\alpha} V^\sigma - \Gamma^\mu_{\rho\alpha,\beta} V^\rho + \Gamma^\mu_{\rho\alpha} \Gamma^\rho_{\sigma\beta} V^\sigma\right]$$

$$= \delta a \cdot \delta b \cdot \left[\Gamma^\mu_{\rho\beta,\alpha} - \Gamma^\mu_{\rho\alpha,\beta} + \Gamma^\mu_{\sigma\alpha} \Gamma^\sigma_{\rho\beta} - \Gamma^\mu_{\sigma\beta} \Gamma^\sigma_{\rho\alpha}\right] V^\rho$$

$$\boxed{\delta V^\mu = \delta a \cdot \delta b \cdot R^\mu_{\rho\alpha\beta} V^\rho}$$

Same R.

$$\boxed{\delta V^\mu = R^\mu_{\rho\alpha\beta} V^\rho \underline{\Delta x}^\alpha \underline{\Delta x}^\beta} \quad (3.109).$$



$$x'^\alpha = x^\alpha + \xi^\alpha$$

$\lambda = \lambda_0$ $\quad \frac{d\hat{x}'}{d\lambda} = \frac{d\hat{x}}{d\lambda} = \hat{u} \quad (\hat{u}'_0 = \hat{u}_0)$

geodesics. $\nabla_{u'}\hat{u}' = 0 \qquad \frac{d^2x'^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}u'^\alpha u'^\beta = 0$

$\nabla_u \hat{u} = 0 \qquad \frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta = 0$

$$[\hat{n}, \hat{u}] = \nabla_n \hat{u} - \nabla_u \hat{n}$$

$$= n^\alpha\left(\frac{\partial u^\mu}{\partial x^\alpha} + \Gamma^\mu_{\lambda\alpha}u^\lambda\right) - u^\alpha\left(\frac{\partial n^\mu}{\partial x^\alpha} + \Gamma^\mu_{\lambda\alpha}n^\lambda\right) \to 0.$$

$$= \frac{du^\mu}{dn} - \frac{dn^\mu}{d\lambda} = 0$$ initial setup

$$\nabla_u(\nabla_u \hat{n}) = \nabla_u(\nabla_n \hat{u})$$

$$= \nabla_u[\underbrace{\cancel{\nabla_u u}}_{\text{geodesic}}] + [\nabla_n, \nabla_u]u$$

$$\boxed{(\nabla_u \nabla_u n)^\mu = R^\mu{}_{\alpha\beta\gamma}u^\alpha u^\beta n^\gamma}$$ Schutz (687)

$$\frac{D^2 n^\mu}{D\lambda^2} = R^\mu{}_{\alpha\beta\gamma}u^\alpha u^\beta n^\gamma \qquad (3.208).$$

$u \to T \qquad \frac{d}{d\lambda} \to \frac{d}{dt}$

Contractions.

$$\boxed{R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}}$$ Ricci tensor. $R_{\mu\nu} = R_{\nu\mu}$

(other choices are either $\pm$, or vanish.)

$$\boxed{R = R^{\mu}{}_{\mu} = g^{\mu\nu} R_{\mu\nu}}$$ Ricci scalar. curvature scalar.

$R_{\rho\sigma\mu\nu}$ $\qquad \frac{1}{12} n^2(n^2-1)$

$R_{\mu\nu}$ $\qquad \frac{1}{2} n(n+1)$

R $\qquad 1.$

$n=1$ $\quad R_{\mu\nu\rho\sigma} = R_{\mu\nu} = R = 0.$

$n=2$ $\quad \frac{1}{2} n(n+1) = 1 = \frac{1}{12} n^2(n^2-1).$ $\qquad \underline{R_{0101}}$

$$\underline{R_{\rho\sigma\mu\nu} = \tfrac{1}{2}(g_{\rho\mu} g_{\sigma\nu} - g_{\rho\nu} g_{\sigma\mu}) R}$$

$$R_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma} \cdot (g_{\rho\sigma} \cdot g_{\mu\nu} - g_{\rho\nu} g_{\sigma\mu}) R$$
$$= \tfrac{1}{2}(2 \cdot g_{\mu\nu} - g_{\mu\nu}) R = \underline{\tfrac{1}{2} g_{\mu\nu} R}$$

$n=3$ $\quad \frac{1}{2} n(n+1) = 6$

$\frac{1}{12} n^2(n^2-1) = 6$

$$R_{\rho\sigma\mu\nu} = 2\left(g_{\rho[\mu} R_{\nu]\sigma} - g_{\sigma[\mu} R_{\nu]\rho}\right)$$