10/17/2016 Gravity everybody has same motion in gravitational field.

$$\vec{F} = m\vec{a} = -\frac{GMm}{r^2}\hat{r} \qquad \boxed{\frac{d^2\vec{x}}{dt^2} = -\vec{\nabla}\phi}$$

Dicke, Roll, Krotkov 1964 "Eötvös" experiment.

$\boxed{\eta(Au, Al) < 3\times10^{-11}}$ 95% CL.

$\eta = (1.3 \pm 1.0)\times10^{-11}$

Au. $A=79$ $M=196.966$ 0.473

Al, $A=13$ $M=26.98$ 0.481

"fifth force. $\frac{\Delta a}{a}$ towards sun

→ everybody following geodesic

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

choose $\boxed{\lambda = \tau}$ (can)

in neighborhood. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ small.

$$\frac{dx^\mu}{d\tau} = \left(\frac{\gamma}{\gamma\vec{v}}\right) \simeq \left(\frac{1}{v}\right) + O(v^2) \qquad \left(\frac{30\text{ km/s}}{300{,}000\text{ km/s}} = 10^{-4}\right)$$

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00} = 0.$$

Γh^{ij} ②

$$\Gamma^i{}_{00} = \frac{1}{2} g^{i\rho}\left(g_{\rho 0,0} + g_{\rho 0,0} - g_{00,\rho}\right)$$

$$\simeq \frac{1}{2}\left(2 g_{i0,0} - g_{00,i}\right) = -\frac{1}{2}\nabla_i (g_{00})$$

$$\boxed{\frac{\partial}{\partial t} \ll \frac{\partial}{\partial x}} \qquad x \sim t \quad \frac{\partial}{\partial t} \sim \frac{\partial}{\partial x}$$

$$\boxed{g_{00} \simeq -(1+2\phi)} \longrightarrow \frac{d^2 x^i}{d\tau^2} \simeq -\nabla_i \phi$$

$\boxed{\frac{\partial}{\partial t} \simeq \text{killing vector}} \longrightarrow p_t$ constant in time

@ $\phi = 0$. $\quad p^\mu = \begin{pmatrix} E_0 \\ \vec{p}_0 \end{pmatrix} = \begin{pmatrix} \omega_0 \\ \vec{k}_0 \end{pmatrix} \qquad u_{obs} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix}$

$$\omega_{obs} = -u \cdot p = -\left(-(1)(\omega_0) + \cancel{\vec{0}\cdot \vec{k}_0}\right) = \omega_0 .$$

@ $\phi = \frac{gh}{c^2} \neq 0$ $\qquad u \cdot u = -1 = -(1+2\phi)(u^0)^2$

$$(u^0)^2 \simeq (1-2\phi) = (1-\phi)^2 \; / \; \boxed{u^0 = 1-\phi}$$

$$\omega_{obs} = -p \cdot u = -\left(-(\omega_0)(1-\phi) + \cancel{\vec{k}\cdot\vec{0}}\right) = \omega_0(1-\phi)$$

$$\boxed{\omega_{obs} = \omega_0(1-\phi) = \omega_0\left(1 - \frac{gh}{c^2}\right)}$$

gravitational redshift.

[(Robert) Pound (Glen) Rebka. 1959 Mössbauer. 10% → 1% (Pound Snider)

3

$$S = -m \int_1^2 d\lambda \left(-g_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} \right)^{1/2} = -m\,\tau_{12}$$

parametrization invariant $\quad d\lambda = \frac{d\lambda}{d\mu} d\mu$

$$\left[\frac{\delta S}{\delta x^\mu(\lambda)} = 0 \right] \longrightarrow -\frac{d}{d\lambda} \frac{\partial L}{\partial x^{\mu\prime}} + \frac{\partial L}{\partial x^\mu} = 0$$

$$-\frac{d}{d\lambda} \left[\frac{1}{2} (\cdots)^{-1/2} \cdot 2 g_{\mu\sigma} \frac{dx^\sigma}{d\lambda} \right]$$

$$+ \frac{1}{2} (\cdots)^{-1/2} \left(-g_{\rho\sigma,\mu} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} \right) = 0$$

choose: $\lambda = \tau \qquad \left(-g_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} \right)^{1/2} = \left(-u \cdot u \right)^{1/2} = 1.$

$$\frac{d}{d\tau} \left(g_{\mu\sigma} \frac{dx^\sigma}{d\tau} \right) + \frac{1}{2} g_{\rho\sigma,\mu} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

$$\frac{d^2 x^\sigma}{d\tau^2} g_{\mu\sigma} + \frac{dx^\sigma}{d\tau} \cdot g_{\mu\sigma,\lambda} \frac{dx^\lambda}{d\tau} + \frac{1}{2} g_{\rho\sigma,\mu} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

Conformal. $ds^2 = e^{2\Phi} \cdot \eta_{\mu\nu} dx^\mu dx^\nu$.

$$S = -m \int d\tau \left(-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right)^{1/2}$$

$$= -m \int d\tau \; e^{\Phi} \left(-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right)^{1/2}$$

Euler-Lagrange $\dot{x}^\mu$.

$$\frac{d}{d\tau}\left[e^{\Phi} \cdot \tfrac{1}{2}(\ldots)^{-1/2} \left(-\eta_{\mu\nu} \cdot 2 \frac{dx^\nu}{d\tau}\right)\right]$$

$$= \frac{\partial}{\partial x^\mu}\left(e^{\Phi}(\ldots)^{1/2}\right)$$

$$-\frac{d}{d\tau}\left(e^{2\Phi} \eta_{\mu\nu} \frac{dx^\nu}{d\tau}\right) = \frac{\partial \Phi}{\partial x^\mu} \cdot \left(\underbrace{e^{\Phi}(\ldots)^{1/2}}_{1}\right).$$

$\eta^{\mu\sigma}$ $\quad \delta^\sigma_\nu$

$$\left\{ e^{2\Phi} \eta_{\mu\nu} \frac{d^2x^\nu}{d\tau^2} + \eta_{\mu\nu} \frac{dx^\nu}{d\tau} \frac{d}{d\tau}\left(e^{2\Phi}\right) = -\frac{\partial \Phi}{\partial x^\mu} \right\}$$

$$e^{2\Phi} \frac{d^2x^\sigma}{d\tau^2} + \delta^\sigma_\nu \frac{dx^\nu}{d\tau} \, 2 \frac{\partial \Phi}{\partial x^\rho} \frac{dx^\rho}{d\tau} e^{2\Phi} = -\frac{\partial \Phi}{\partial x^\mu} \cdot \eta^{\mu\sigma}$$

$$e^{2\Phi} \frac{d^2x^\sigma}{d\tau^2} + e^{2\Phi} \frac{\partial \Phi}{\partial x^\rho} u^\rho u^\sigma = -\frac{\partial \Phi}{\partial x^\rho} \cdot \eta^{\rho\sigma}.$$

$$\boxed{\frac{d^2x^\sigma}{d\tau^2} = -\frac{\partial \Phi}{\partial x^\rho}\left(\eta^{\rho\sigma} e^{-2\Phi} + u^\rho u^\sigma\right)}$$