10/21/2016

$G_{\mu\nu} = 8\pi G T_{\mu\nu}$.

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$.

$\rightarrow \boxed{\Box^2 \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}}$

$\bar{h}_{\mu\nu,}{}^{\nu} = 0$

$T_{\mu\nu} = \begin{pmatrix} \rho & \\ & 0 \end{pmatrix}$ $\qquad g_{\mu\nu} = \left(\begin{array}{c|c} -(1+2\phi) & \\ \hline & (1-2\phi)\delta_{ij} \end{array}\right)$

$\uparrow$

$(1-2\gamma)\phi$

$\Gamma^{\mu}{}_{\alpha\beta} = \frac{1}{2}\eta^{\mu\gamma}\left(h_{\gamma\alpha,\beta} + h_{\gamma\beta,\alpha} - h_{\alpha\beta,\gamma}\right)$

$\Gamma^{0}{}_{\alpha\beta} = \frac{1}{2}(-)\left(h_{0\alpha,\beta} + h_{0\beta,\alpha} - \cancel{h_{\alpha\beta,0}}\right)$

$\alpha = 0 \quad \beta = i$

$\alpha = i \quad \beta = 0$

$h_{00,i}$

$\boxed{\Gamma^{0}{}_{0i} = \Gamma^{0}{}_{i0} = \left(\tfrac{1}{2}\right)(-)(-2\phi)_{,i} = \phi_{,i}}$

$\Gamma^{i}{}_{\alpha\beta} = \frac{1}{2}(+)\left(h_{i\alpha,\beta} + h_{i\beta,\alpha} - h_{\alpha\beta,i}\right)$

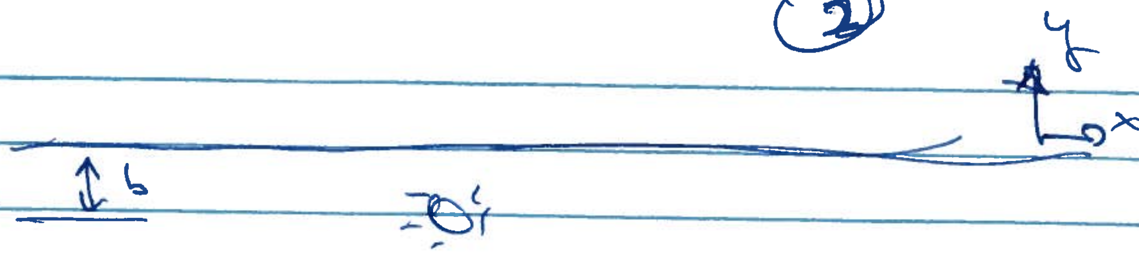
$\alpha = i,\ \beta = j \qquad \beta = i,\ \alpha = j \qquad \alpha = \beta = 0$.

$\Gamma^{i}{}_{00} = \phi_{,i}$

$\cancel{\Gamma^{i}{}_{jk} = \delta_{ij}\phi_{,k} + \delta_{ik}\phi_{,j}}$ $\quad \Gamma^{2}{}_{11} = (\gamma)\phi_{,y}$

$\cancel{\Gamma^{i}{}_{ij} = \Gamma^{i}{}_{ji} = \phi_{,j}}$ $\quad \Gamma^{2}{}_{11} = \frac{1}{2}(+)\left(\cancel{h_{12,1}} + \cancel{h_{12,1}} - h_{11,2}\right)$

$\Gamma^{1}{}_{1k} = \frac{1}{2}(+)\left(h_{11,k} + \cancel{h_{1k,1}} - h_{1k,1}\right) = -\phi_{,k}$



deflection small → (perturbation theory)

undeflected: $x = t$. $y = b$

$\frac{d^2x^\mu}{d\lambda^2} = \frac{du^\mu}{d\lambda}$ = constant $\quad u^t = \frac{dt}{d\lambda} = 1 \quad \frac{dx}{d\lambda} = 1 = u^x$

$t(\lambda) = \lambda \qquad u^t = 1$

$x(\lambda) = \lambda \qquad u^x = 1$

$y(\lambda) = b \qquad u^y = 0$

Turn on sun: $\phi = -\frac{GM}{r}$ $\qquad \nabla^2 h = 0$

$$\boxed{\frac{du^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0}$$

$\alpha = 0$ or 1

$\beta = 0$ or 1.

$$\frac{du^\mu}{d\lambda} = -\left(\Gamma^\mu_{00} + 2\Gamma^\mu_{01} + \Gamma^\mu_{11}\right).$$

$$\frac{du^y}{d\lambda} = -\left(\Gamma^y_{00} + \Gamma^y_{xx} + 2\cancel{\Gamma^y_{0x}}\right)$$

diagonal metric

$$= -\left(\phi_{,y} + \gamma\phi_{,y}\right) = (1+\gamma)\left(-\frac{GMy}{r^3}\right)$$

$$\frac{du^y}{dx} = -(1+\gamma)\frac{Gm\,b}{(x^2+b^2)^{3/2}}$$

$$\Delta u^y = \int_{-\infty}^{\infty}\frac{du^y}{dx}\,dx = -(1+\gamma)\int_{-\infty}^{\infty}\frac{Gmb}{(x^2+b^2)^{3/2}}$$

$$= -(1+\gamma)\frac{2Gm}{b}$$

$$\boxed{\Delta\theta = \frac{\Delta p}{p} = (1+\gamma)\left(\frac{2Gm}{bc^2}\right)}$$

$m = 2\times10^{33}\,g.$ $\quad \boxed{\frac{2Gm}{c^2} = 1.5\ \text{km}}$

$b = R_\odot = 696{,}000$ km.

$\delta\theta = (1+\gamma)(0.875\ \text{arcsec})$

$\gamma = 2$ $\quad \boxed{1.75\ \text{arcsec}}$

~~Nordström~~

~~Newton~~.

Eddington 1919.

Shapiro et al. 2004. PRL $\quad \boxed{\gamma = 0.99983 \pm 0.00043}$

2-6

$$\Delta t = 2(1+\gamma)\frac{Gm}{bc^2}\ln\left(\frac{2r_1r_2}{b^2}\right)$$

$b \ll r$

$$\boxed{\gamma = 1 + (2.1 \pm 2.3)\times10^{-5}}$$

$\delta_\beta = (-2.1 \pm 2.3)\times10^{-5}$

$1 - \delta_\beta = 1 + (2.1 \pm 2.3)\times10^{-5}$

$\frac{1+\gamma}{2} = \frac{1+\gamma_\beta}{2}$

Perihelion precession. need new formulation.

Spherical Symmetry. Ch. 5

Flat space. $ds^2 = -dt^2 + dr^2 + r^2 \underbrace{(d\theta^2 + \sin^2\theta\, d\phi^2)}_{d\Omega^2}$

Spherically symmetric, static.

$$ds^2 = -e^{2\alpha} dt^2 + e^{2\beta} dr^2 + e^{2\gamma} r^2 d\Omega^2$$

not really flat — what does "r" mean.

might like: $+ \cdots + \cdots \hat{dr}^2 + \cdots$

$e^{2\beta} dr^2 \rightarrow \cancel{\hat{dr}^2}$ doesn't work.

Can do: $e^{2\gamma} r^2 = \bar{r}^2$. $\bar{r} = r e^{\gamma}$

$$d\bar{r} = \left(1 + r\frac{d\gamma}{dr}\right) dr \cdot e^{\gamma}$$

$$ds^2 = -e^{2\alpha} dt^2 + \frac{e^{2\beta - 2\gamma}}{\left(1 + r\frac{d\gamma}{dr}\right)^2} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

rename $\bar{r} \rightarrow r$

$g_{rr} \rightarrow e^{2\beta}$.

5

$$\boxed{ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2 d\Omega^2}$$

r = "Schwarzschild" r

(surface of constant r, constant t, has proper surface area $\underline{A = 4\pi r^2}$)

Non-zero Γ's: need (tt), (rr), $(\theta\theta)$, $(\phi\phi)$ + r or $(\phi\phi)$ + θ

$$\Gamma^r_{tt} = \frac{1}{2} g^{rr}\left(\cancel{g_{rt,t}} + \cancel{g_{rt,t}} - g_{tt,r}\right)$$

$$= \frac{1}{2}\left(e^{-2\beta}\right)\left(-2\alpha_{,r}\, e^{2\alpha}\right)$$

$$\boxed{\Gamma^r_{tt} = \alpha_{,r}\, e^{2(\alpha-\beta)}}$$

$$\boxed{\Gamma^t_{tr} = \Gamma^t_{rt} = \alpha_{,r}} \qquad \boxed{\Gamma^r_{rr} = \beta_{,r}}$$

$$\boxed{\Gamma^\theta_{\theta r} = \Gamma^\phi_{\phi r} = \frac{1}{r}} = \gamma_{,r} \quad (\gamma = \log r)$$

$$\boxed{\Gamma^r_{\theta\theta} = -r e^{-2\beta}} \qquad \boxed{\Gamma^r_{\phi\phi} = -r\sin^2\theta\, e^{-2\beta}}$$

$$\boxed{\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta} \qquad \boxed{\Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta}}$$

as before

6

$$R^{tr}{}_{tr} = e^{-2\beta}\left[-\alpha_{,rr} - (\alpha_{,r})^2 + \alpha_{,r}\beta_{,r}\right] \qquad e^{-2\beta}$$

$$R^{t\theta}{}_{t\theta} = -\frac{1}{r}e^{-2\beta}\alpha_{,r} \qquad \frac{1}{r^2}$$

$$R^{t\phi}{}_{t\phi} = -\frac{1}{r}e^{-2\beta}\alpha_{,r} \qquad \frac{1}{r^2\sin^2\theta}$$

$$R^{r\theta}{}_{r\theta} = \frac{1}{r}e^{-2\beta}\beta_{,r} \qquad \frac{1}{r^2}$$

$$R^{r\phi}{}_{r\phi} = \frac{1}{r}e^{-2\beta}\beta_{,r} \qquad \frac{1}{r^2\sin^2\theta}$$

$$R^{\theta\phi}{}_{\theta\phi} = \frac{1}{r^2}\left(1-e^{-2\beta}\right) \qquad \frac{1}{r^2\sin^2\theta}$$

$$R^{t\theta}{}_{r\theta} = -\frac{1}{r}e^{-2\beta}\alpha_{,r}$$

$$R^{t\phi}{}_{r\phi} = -\frac{1}{r}e^{-2\beta}\alpha_{,r}$$

$$R^{tr}{}_{r\theta} = R^{tr}{}_{r\phi} = R^{tr}{}_{t\theta} = R^{tr}{}_{t\phi}$$

$$= R^{t\theta}{}_{\theta\phi} = R^{r\theta}{}_{\theta\phi} = R^{t\phi}{}_{\phi\theta} = R^{r\phi}{}_{\phi\theta} = 0$$

$$R_{tr\theta\phi} = 0$$