

10/28/2016

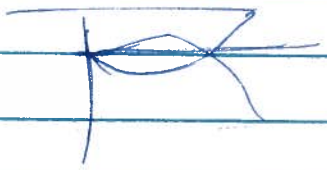
$$\left(\frac{dr}{dt}\right)^2 + \left(1 - \frac{2M}{r}\right)\left(1 + \frac{L^2}{r^2}\right) = \tilde{E}^2$$

$\frac{d}{dt}$

$u = \frac{1}{r}$

\rightarrow

$A + Bx^2 - x$



$$\frac{d^2 u}{d\phi^2} + u = \frac{M}{L^2} + 3Mu^2$$

circle

$u = u_0$

$$u_0 = \frac{M}{L^2} + 3Mu_0^2$$

$\approx \frac{M}{L^2}$

perturb.

$u = u_0 + \Delta u$

$$\frac{d^2}{d\phi^2}(\Delta u) + u_0 + \Delta u = \frac{M}{L^2} + 3M(u_0 + \Delta u)^2$$

$$\frac{d^2}{d\phi^2}(\Delta u) + (1 - 6Mu_0)\Delta u = 0$$

$$\Delta u = u_1 \cos \phi$$

$\alpha^2 = 1 - 6Mu_0$

$\alpha = 1 - 3Mu_0$

$u = u_0 + u_1 \cos \phi$

$u = \frac{1 + e \cos \phi}{a(1 - e^2)}$

$$3Mu_0 = \frac{3M/c^2}{a(1 - e^2)} = \frac{3(1.477 \text{ km})}{(0.38709)(149.6 \text{ M km})}$$

$\times (1 - 0.2056^2)$

$= 7.989 \times 10^{-8}$

(century) = 415.2 orbits $(415)(8.10^{-8})(360)(60)(60)$
 (87.97d) = 42.99 arcsec

Periodic → harmonics: $u = u_0 + u_1 \cos \phi + u_2 \cos 2\phi$

$$u = \sum_{n=0}^{\infty} u_n \cdot \cos n\phi$$

$$u'' + \mu u = \sum_{n=0}^{\infty} (1 - n^2 \alpha^2) u_n \cos n\phi$$

$$= \frac{\mu}{L^2} + 3M u^2 = \frac{\mu}{L^2} + 3M \left(\sum_n u_n \cos n\phi \right)^2$$

$$\cos nx \cos mx = \frac{1}{2} (\cos(n+m)x + \cos(n-m)x)$$

$$u_0 = \frac{\mu}{L^2} + 3M \left(u_0^2 + \sum_{n=1}^{\infty} \frac{1}{2} u_n^2 \right)$$

$$(1 - \alpha^2) u_1 = 3M \left(2u_0 u_1 + \sum_{n=1}^{\infty} u_n u_{n+1} \right)$$

$$(1 - 4\alpha^2) u_2 = 3M \left(2u_0 u_2 + \frac{1}{2} u_1^2 + \sum_{n=1}^{\infty} u_n u_{n+2} \right)$$

$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$

0, 1, 2

$$u_0 = \frac{\mu}{L^2} + 3M u_0^2 + \frac{3}{2} M u_1^2$$

$$(1 - \alpha^2 - 6M u_0) u_1 = 3M u_1 u_2$$

$$(1 - 4\alpha^2 - 6M u_0) u_2 = \frac{3}{2} M u_1^2$$

$$1 - \alpha^2 - 6M u_0 = 0 \quad \alpha^2 = 1 - 6M u_0$$

$$u_0 = \frac{1}{a(1-e^2)}$$

$$u_1 = \frac{e}{a(1-e^2)}$$

$$\frac{3}{2} \frac{M e^2}{a^2 (1-e^2)^2}$$

Small

Binary Pulsar, Taylor & Hulse PSR B1501-10.

$$a \sin i = 2.34185 \text{ sec.}$$

$$\sin i = 0.76 \rightarrow 0.14$$

$$e = 0.617127$$

$$P = 27,906 \text{ sec.} = 7.75 \text{ h.}$$

$$M_1, M_2 = 1.4 M_\odot$$

$$\frac{3GM}{a^3(1-e^2)} = \frac{3(0.7)(1.5 \text{ km})}{(703000 \text{ km})(120 \text{ km})(0.617)} = \frac{7 \times 10^{-6}}{7.2 \times 10^6}$$

$$\text{Year} = 1130 \text{ P.} \quad 7 \times 10^{-6} \cdot 1130 \cdot 360^\circ = 2.94 \text{ degrees}$$

$$\text{or } 4.22 \text{ deg. yr}^{-1}$$

SO-2, $P = 14.534$

$$A = 919 \text{ AU}$$

$$e = 0.8670$$

$$M = 3.67 \times 10^6 M_\odot$$

$$\sqrt{2} = \frac{2\pi^2}{P} = \frac{M}{r^3}$$

$$\frac{3GM}{A(1-e^2)} = \frac{3(3.67 \times 10^6)(1.5 \text{ km})}{(919)(150 \text{ km})(1-0.75)}$$

$$= 4.8 \times 10^{-4}$$

$$\Rightarrow \langle \dot{\omega} \rangle = 10 \text{ arcmin per orbit.}$$

$$6 \text{ orbits} = 814 \rightarrow 1 \text{ degree.}$$