10/28/2016

$$\left(\frac{dr}{d\tau}\right)^2 + \left(1-\frac{2M}{r}\right)\left(1+\frac{\tilde{L}^2}{r^2}\right) = \tilde{E}^2$$

$\frac{d}{d\phi}$ $\quad u = \frac{1}{r} \longrightarrow$

$A + Bx^2 - x$

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{\tilde{L}^2} + 3Mu^2$$

circular $\quad u = u_0 \qquad u_0 = \frac{M}{\tilde{L}^2} + 3Mu_0^2 \approx \frac{M}{\tilde{L}^2}$.

perturb. $\quad u = u_0 + \Delta u$.

$$\frac{d^2}{d\phi^2}(\Delta u) + u_0 + \Delta u = \frac{M}{\tilde{L}^2} + 3M(u_0^2 + 2u_0\Delta u)$$

$$\frac{d^2}{d\phi^2}(\Delta u) + (1 - 6Mu_0)\Delta\phi = 0$$

$\Delta u = u_1 \cdot \cos \alpha\phi \qquad \alpha^2 = 1 - 6Mu_0$

$\alpha = 1 - 3Mu_0$.

$u = u_0 + u_1 \cos \alpha\phi \qquad u = \frac{1 + e\cos\alpha\phi}{a(1-e^2)}$

$$3Mu_0 = \frac{3GM/c^2}{a(1-e^2)} = \frac{3(1.477\ \text{km})}{(0.38709)(149.6\ \text{M km})} \times (1 - 0.2056^2)$$

$$= 7.989 \times 10^{-8}$$

$\frac{\text{(century)}}{(87.97\text{d})} = 415.2$ orbits $\qquad (415)(8.10^{-8})(360)(60)(60) = 42.99$ arcsec

Periodic → harmonics. $u = u_0 + u_1 \cos\alpha\phi + u_2 \cos 2\alpha\phi$,

$$u = \sum_{n=0}^{\infty} u_n \cdot \cos n\alpha\phi$$

$$u'' + u = \sum_{n=0}^{\infty} (1 - n^2\alpha^2)\, u_n \cos n\alpha\phi$$

$$= \frac{u}{L^2} + 3Mu^2 = \frac{u}{L^2} + 3M\left(\sum_n u_n \cos n\alpha\phi\right)^2$$

$$\cos nx \cos mx = \frac{1}{2}\left(\cos(m+n)x + \cos(m-n)x\right)$$

$$u_0 = \frac{u}{L^2} + 3M\left(u_0^2 + \sum_{n=1}^{\infty} \frac{1}{2} u_n^2\right)$$

$$(1-\alpha^2)\, u_1 = 3M\left(2u_0 u_1 + \sum_{n=1}^{\infty} u_n u_{n+1}\right)$$

$$(1-4\alpha^2)\, u_2 = 3M\left(2u_0 u_2 + \frac{1}{2}u_1^2 + \sum_{n=1}^{\infty} u_n u_{n+2}\right)$$

0, 1, 2

$$u_0 = \frac{u}{L^2} + 3Mu_0^2 + \frac{3}{2}Mu_1^2 \qquad u_0 = \frac{1}{a(1-e^2)}$$

$$(1 - \alpha^2 - 6Mu_0)\, u_1 = 3M u_1 u_2 \qquad u_1 = \frac{e}{a(1-e^2)}$$

$$(1 - 4\alpha^2 - 6Mu_0)\, u_2 = \frac{3}{2}M u_1^2 \leftarrow \frac{3}{2}\frac{Me^2}{a^2(1-e^2)^2}$$

Small

$$1 - \alpha^2 - 6Mu_0 \simeq 0 \qquad \alpha^2 = 1 - 6Mu_0$$

Binary Pulsar: Taylor & Hulse PSR 1913+16.

$a \sin i = 2.34185$ sec. $\sin i = 0.76 \pm 0.14$

$e = 0.617127$

$P = 27,906$ sec. $= 7.75$ h. $m_1 \approx m_2 = 1.4\ M_\odot$

$\mu = \frac{1}{2} m$

$$\frac{3GM}{ac^2(1-e^2)} = \frac{3(0.7)(1.5\,km)}{(703000\,km)(1-0.619)} = 7 \times 10^{-6}$$

7.2×10^{-6}

Year $= 1130\,P$. $7 \times 10^{-6} \cdot 1130 \cdot 360^\circ = 2.94$ degrees

obs. $4.22\ deg \cdot yr^{-1}$

SO-2. $P = 14.534$ $A = 919$ AU $e = 0.8670$

$M = 3.67 \times 10^6\ M_\odot$.

$$\Omega^2 = \frac{2\pi^2}{(P)} = \frac{M}{r^3}$$

$$\frac{3GM}{A(1-e^2)} = \frac{3(3.67 \times 10^6)(1.5\,km)}{(919)(150\,M\,km)(1-0.75^2)}$$

$= 4.8 \times 10^{-4}$

$\Delta\phi(\omega_p) = 10$ arcmin per orbit.

6 orbits $= 874 \rightarrow 1$ degree.