11/14/16

GW energy-momentum:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$$

$$G_{\mu\nu} = \cancel{G^{(0)}_{\mu\nu}} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$$

to 2nd order ~~absent vacuum~~

$$= 8\pi G \left(\cancel{T^{(0)}_{\mu\nu}} + T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu}\right)$$

$$\boxed{G^{(1)}_{\mu\nu} = 8\pi G\, T^{(1)}_{\mu\nu}} \leftarrow \text{(possibly} = 0\text{)}.$$

$$G^{(0)}_{\mu\nu} = 8\pi G \left(T^{(0)}_{\mu\nu} + t^{\text{eff}}_{\mu\nu}\right)$$

$$t^{\text{eff}}_{\mu\nu} = -\frac{1}{8\pi G} G^{(2)}_{\mu\nu}$$

$$-8\pi G\, t^{\text{eff.}}_{\alpha\beta} = R^{(2)}_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}\, \eta_{\mu\nu} R^{(2)\mu\nu}$$

$$+ \frac{1}{2}\left(\eta_{\mu\nu} h^{\alpha\beta} - \eta^{\alpha\beta} h_{\mu\nu}\right) R^{(1)\mu\nu}$$

$$+ \cancel{\frac{1}{2}[R\ \ldots]} \to 0 \text{ vacuum.}$$

$$R^{(1)}_{\mu\nu} = \cancel{2\eta_{\mu\nu}}\ \frac{1}{2}\left(-h_{,\mu\nu} - h_{\mu\nu,\alpha}{}^{\alpha} + h_{\alpha\mu,\nu}{}^{\alpha} + h_{\alpha\nu,\mu}{}^{\alpha}\right)$$

$$R^{(2)}_{\mu\nu} = \ldots\ldots$$

2

Landau - Lifschitz . $t^{\alpha\beta}$ "pseudo tensor".

$$16\pi G \boxed{g}\, t^{\alpha\beta} = \underline{\underline{g}}^{\alpha\beta}{}_{,\lambda}\, \underline{\underline{g}}^{\lambda\mu}{}_{,\mu} - \underline{\underline{g}}^{\alpha\lambda}{}_{,\lambda}\, \underline{\underline{g}}^{\beta\mu}{}_{,\mu}$$

$$+ \tfrac{1}{2}\eta^{\alpha\beta}\eta_{\lambda\mu}\, \underline{\underline{g}}^{\lambda\nu}{}_{,\rho}\, \underline{\underline{g}}^{\rho\mu}{}_{,\nu}$$

$$-(\qquad) + (\quad) + \tfrac{1}{8}(\qquad).$$

$$\underline{\underline{g}}^{\alpha\beta} = \sqrt{g}\, g^{\alpha\beta}$$

$$H^{\alpha\mu\beta\nu} = \underline{\underline{g}}^{\alpha\beta}\underline{\underline{g}}^{\mu\nu} - \underline{\underline{g}}^{\alpha\nu}\underline{\underline{g}}^{\beta\mu}$$

$$H^{\alpha\mu\beta\nu}{}_{,\mu\nu} = 16\pi\sqrt{g}\,(T+t)$$

In general . different
not conserved,
"clumsy"

smooth over L
& vanishes at ∞.

"Brill - Hartle average" $\int d^4x.\ H(x')f(|x-x'|) = \langle H\rangle$

$$\langle H_{,\sigma}\rangle \to \left(\frac{\lambda}{L}\right) \to 0.$$

$$\langle h_{\mu\nu,\alpha\beta}\, h^{\gamma\delta}\rangle = \langle -h_{\mu\nu,\alpha}\, h^{\gamma\delta,\beta}\rangle$$

$$\langle h_{\mu\nu,\alpha\beta}\rangle = \langle h_{\mu\nu,\beta\alpha}\rangle$$

$$\boxed{\langle t^{\alpha\beta} \rangle^{GW} = \frac{1}{32\pi G} \langle h^{\rho\sigma,\alpha} h_{\rho\sigma}{}^{,\beta} \rangle}$$

3

Many Sources

$$h_{ij}^{TT} = \left(h_+ A_{+ij} + h_\times A_{\times ij} \right) e^{i(\vec{k}\cdot\hat{x} - \omega t)}$$

$$\langle t^{\alpha\beta} \rangle = \frac{1}{32\pi G} k^\alpha k^\beta \cdot \langle (h_+ A_{+ij} + h_\times A_{\times ij})(h^+ A_+^{ij} + h^\times A^{ij}_\times) \rangle$$

$A^+_{ij} A^+_{ij} = 2$

$A^+_{ij} A^\times_{ij} + A^\times_{ij} A^+_{ij} \to 0$

$$= \frac{1}{32\pi G} k^\alpha k^\beta \left(|h_+|^2 + |h_\times|^2 \right) \times \frac{1}{2}$$

(7.171)

$$\boxed{T_{oz} \sim 10^{-4} \left(\frac{f}{Hz} \right)^2 \frac{(h_+^2 + h_\times^2)}{(10^{-21})^2} \ \frac{erg}{cm^2 \cdot s}}$$

Carroll (7.173).

Sources:

$$\Box^2 \bar{h}_{\mu\nu} = -16\pi G\, T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = \int d^3x' \left(\frac{1}{4\pi} \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \right) (16\pi G\, T_{\mu\nu})$$

$$\bar{h}_{ij} \longrightarrow \frac{e^{ikr}}{r} \cdot 4G \cdot \int d^3x'\, \cancel{e^{-ik\hat{r}\cdot\vec{x}'}}\, T_{ij}.$$

long wavelengths. $kd \ll 1$. $e^{ikd} \simeq 1$.

$$\bar{h}_{ij} \simeq 4\frac{e^{ikr}}{r} \int d^3x'\, T_{ij}$$

Tensor virial theorem: $\int d^3x\, T_{ij} = \frac{1}{2}\frac{d^2}{dt^2}\int d^3x'\, x'^i x'^j\, T^{00}$

$$= -\frac{1}{2}\omega^2 I_{ij}$$ Quadrupole moment of inertia

$$\boxed{\bar{h}_{ij} = -2\omega^2 \frac{e^{ikr}}{r} I_{ij}} \quad (7.139)$$

$$\langle P \rangle = \frac{1}{16\pi G} (2\omega^2)^2 \cdot \frac{1}{2}(r^2) \int \overset{TT}{I_{ij}}\, \overset{TT}{I_{ij}}^{*}\, d\Omega.$$