11/16/2006

$$\bar{h}_{ij} = -2\omega^2 \frac{e^{ikr}}{r} G \bar{I}_{ij}^{TT} \qquad (7.139)$$

$$t^{\mu\nu} = \frac{1}{32\pi G} \langle h_{ij}{}^{,\mu} h_{ij}{}^{,\nu} \rangle$$

$k^0 = \omega$, $k^i = \omega \hat{r}$

$$\langle P \rangle = \int r^2 d\Omega \, \langle \hat{r} \cdot \vec{S} \rangle = \int r^2 d\Omega \, \frac{\hat{r}\cdot}{32\pi G} \omega^2 \hat{r} \langle \bar{h}_{ij}^{TT} \bar{h}_{ij}^{TT} \rangle$$

$\frac{1}{2} \mathrm{Re}\, h h^*$

$$h_{ij}^{TT} = P \cdot h \cdot P - \frac{1}{2} P \left(\mathrm{Tr}(P \cdot h) \right)$$

$$h_{ij} h_{ij} = \left[(\delta_{im} - \hat{r}_i \hat{r}_m)(\delta_{jn} - \hat{r}_j \hat{r}_n) - \frac{1}{2}(\delta_{ij} - \hat{r}_i \hat{r}_j)(\delta_{mn} - \hat{r}_m \hat{r}_n) \right]$$

$$\left[(\delta_{ip} - \hat{r}_i \hat{r}_p)(\delta_{jq} - \hat{r}_j \hat{r}_q) - \frac{1}{2}(\delta_{ij} - \hat{r}_i \hat{r}_j)(\delta_{pq} - \hat{r}_p \hat{r}_q) \right]$$

$\bar{I}_{mn} \bar{I}_{pq}$

$$= \left((\delta_{mp} - \hat{r}_m \hat{r}_p)(\delta_{nq} - \hat{r}_n \hat{r}_q) - 2 \cdot \frac{1}{2} \cancel{(\delta_{mn} - \hat{r}_m \hat{r}_n)(\delta_{pq} - \hat{r}_p \hat{r}_q)} \right.$$

$$\left. + \frac{1}{4} \cdot 2 \cdot (\delta_{mn} - \hat{r}_m \hat{r}_n)(\delta_{pq} - \hat{r}_p \hat{r}_q) \right)$$

$$= \underline{\delta_{mp} \delta_{nq}} - \underline{(\delta_{mp} \hat{r}_n \hat{r}_q + \delta_{nq} \hat{r}_m \hat{r}_p)}$$

$$+ \underline{\hat{r}_m \hat{r}_n \hat{r}_p \hat{r}_q (1 - 1 + \tfrac{1}{2})}$$

2

$$\int d\Omega \cdot 1 = 4\pi$$

$$\int d\Omega \cdot \hat{r}_i \hat{r}_j = \frac{4\pi}{3} \delta_{ij}$$

$$\int d\Omega \; \hat{r}_i \hat{r}_j \hat{r}_k \hat{r}_l = \frac{4\pi}{15} \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right)$$

$$P = \frac{\omega^2}{16\pi} \cdot 4 \left[4\pi \, \delta_{mp}\delta_{nq} - \frac{4\pi}{3} \left(\delta_{mp}\delta_{nq} + \delta_{nq}\delta_{mp} \right) \times 2 \right.$$

$$\left. + \frac{4\pi}{15} \left(\cancel{\delta_{mn}\delta_{pq}} + \delta_{mp}\delta_{nq} + \delta_{mp}\delta_{nq} \right) \cancel{\times 2} \right] \mathcal{I}_{mn} \mathcal{I}^*_{pq}$$

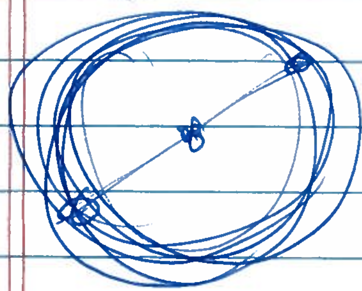
$$\frac{1}{4\pi} \; (2\omega^2)^2 \; \frac{1}{2}$$

$$= \frac{\omega^2}{16\pi} \cdot 4 \cdot 4\pi \cdot \left(1 - 2 \cdot \frac{1}{3} + \frac{2 \cdot}{15} \cdot \frac{1}{2} \right) \cdot \times \frac{1}{2}$$

$$\frac{15 - 10 + 1}{15} = \frac{6}{15} = \frac{2}{5}$$

$$\boxed{P = \frac{\omega^6}{10} \mathcal{I}_{ij} \mathcal{I}^*_{ij}} \qquad \boxed{P = \frac{1}{5} \langle \dddot{\mathcal{I}}_{ij} \dddot{\mathcal{I}}_{ij} \rangle}$$

MTW.



$m_1 = m_2 = m_3$,

$a_c = \omega^2 R = \frac{GM}{(2R)^2}$ $\boxed{\omega^2 = \frac{GM}{4R^3}}$

$I_{ij} = \int d^3x \, (x_i x_j - \delta_{ij} r^2)\rho$.

$I_{xx} = 2MR^2 \cos^2\omega t - \frac{2}{3}MR^2 = MR^2 \cdot \cos 2\omega t + \frac{1}{3}MR^2$

$I_{yy} = 2MR^2 \sin^2\omega t - \frac{2}{3}MR^2 = -MR^2 \cos 2\omega t + \frac{1}{3}MR^2$

$I_{xy} = I_{yx} = 2MR^2 \sin\omega t \cos\omega t = MR^2 \cdot \sin 2\omega t$

$I_{zz} = -\frac{2}{3}MR^2$

$h_{ij} = \frac{2(2\omega)^2 (MR^2)}{r} e^{ikr}$

$I_{ij} = \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} MR^2 e^{-2i\omega t}$

$$\boxed{I^{TT}_{ij} I^{TT*}_{ij} = (MR^2)^2 \left(\tfrac{1}{2}\right)(1 + 6\cos^2\theta + \cos^4\theta)}$$

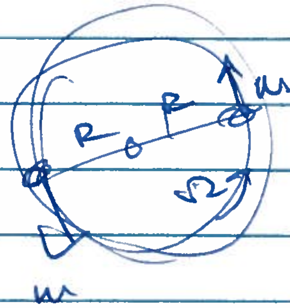
$\frac{1}{16}(35 + 28\cos 2\theta + \cos 4\theta)$

$\frac{1}{32\pi}$

$\frac{32\pi}{15} (MR^2)^2 \frac{(2\cdot(2\omega^2))^2}{\cancel{16\pi} G} \cdot \frac{1}{\cancel{2}} = \frac{64}{15} \frac{\omega^6 (MR^2)^2}{G}$

$\frac{64}{15} \left(\frac{GM}{4R^3}\right)^3 \frac{(MR^2)^2}{G} = \frac{1}{15} \frac{M^5}{R^5}$

$$\frac{d^2}{dt^2} = -\omega^2$$



$$\bar{h}_{ij} = \frac{8 G m \Omega^2 R^2}{r}$$

$$\times \begin{pmatrix} -\cos 2\omega t & -\sin 2\omega t & 0 \\ -\sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & \end{pmatrix}$$

$$\begin{pmatrix} -1 & -i & 0 \\ -i & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{-2i\omega t}$$

$$\boxed{P = -\frac{2}{5}\frac{G^4 m^5}{R^5}}$$

(radiated)

$$G\,(mR^2)^2 \left(\frac{Gm}{R^3}\right)^3$$

$$G I^2 \Omega^6$$

$$I^{TT}_{ij} I^{*TT}_{ij} = 4 \cdot (mR^2)^2$$

Cancell

$$(kd)^2 = \Omega^2 R^2 = \left(\frac{Gm}{R^3}\right) R^2 = \left(\frac{Gm}{R}\right) = \left(\frac{R_s}{R}\right) \ll 1$$

Always "long wavelength"

$m_1 = m_2 = 30\, M_\odot$

$r = (z = \frac{1}{2}) = \frac{1}{2} \cdot cH_0^{-1} = \frac{1}{2} \cdot (14\,Gy) = \boxed{7\,Gy}$

$h = \frac{2\omega^2 G I}{r} = 2\left(\frac{Gm}{R^3}\right)\left(\frac{MR^2}{r}\right)$

$= \frac{M^2}{Rr}$ @ coalescence. $m = R$.

$\frac{M}{R} = \frac{(30 \cdot 3\,km)}{(14 \times 10^9 \times 300{,}000\,km/s \cdot \pi \cdot 10^7)} = \frac{100\,km}{10^{23}\,km}$

$= 10^{-21}$!

$\Omega^2 = \frac{Gm}{4R^3} = \frac{1}{4R^2} = \frac{1}{(2R)^2} = \frac{1}{(200\,km)^2}$

$\Omega = \frac{300{,}000\,km/s}{200\,km} = 1500$ rad. s.

$f = \frac{\Omega}{2\pi} = \cancel{250\,Hz}$. $\boxed{100\,Hz}$

~~R = 3 km~~

35 - 150