11/28/2016 Black Hole: (Schwarzschild).

$$A = 4\pi R^2 = 4\pi(2m)^2 = 16\pi m^2$$

Kerr: (rotating) $a = \frac{J}{m}$

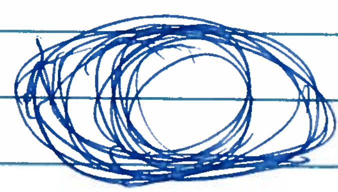
$r_+ = m + \sqrt{m^2 - a^2}$

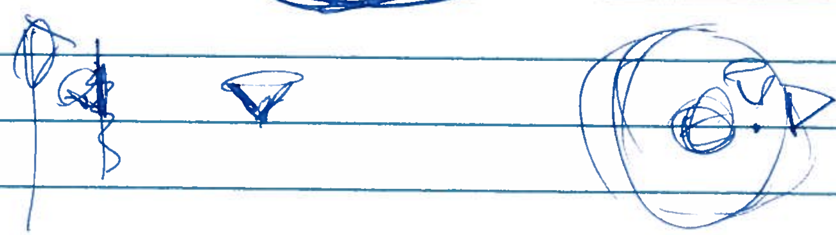
(outer) event horizon

$r_E = m + \sqrt{m^2 - a^2\cos^2\theta}$

Ergosphere

"static limit"





$$\boxed{A = 8\pi M\left(M + \sqrt{M^2 - a^2}\right)}$$

$a < m$

GW150914 $a/m = 0.67$ $m_f > 1.5m$

$A_1 = A_2 = 16\pi M^2$ $m_f^2 \geq \frac{32}{14} m^2 \simeq 2.29 m^2$

$A_f = 8\pi m_f^2\left(1 + \sqrt{1 - (a/m)^2}\right) = 8\pi m_f^2 (1 + 0.8) = \cancel{14.4\pi m_f^2}$ 13.96

$\cancel{0.67}$

$\sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}} = 0.745$ $8 \times \frac{9}{5} = \frac{72}{5} = 14.4$

$$= -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)\,d\phi - a\,dt\right]^2$$

Kerr

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 - \frac{2Mar\sin^2\theta}{\rho^2}\left(d\phi\, dt + dt\, d\phi\right)$$

$$+ \frac{\rho^2}{\Delta^2}dr^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2$$

$$+\rho^2 d\theta^2$$

$$\Delta^2 = r^2 - 2Mr + a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$a \to 0$ Schwarzschild.

$M \to 0$
$$-dt^2 + \frac{r^2 + a^2\sin^2\theta}{r^2+a^2}dr^2$$

$$+ (r^2 + a^2\cos^2\theta)\,d\theta^2 + (r^2+a^2)\sin^2\theta\, d\phi^2$$

Singularity. $(r=0$ and $\cos\theta = 0)$ ring

$$R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} = \frac{48\left(r^2 - a^2\cos^2\theta\right)\left[(r^2+a^2\cos^2\theta)^2 - 16r^2a^2\cos^2\theta\right]}{(r^2+a^2\cos^2\theta)^6}$$

$\kappa$ = surface gravity constant on Horizon.

$$\kappa = \frac{\sqrt{m^2-a^2}}{2m\left(m+\sqrt{m^2-a^2}\right)}$$

$\Delta A \geq 0$

Thermodynamics.

$$T = a \cdot \kappa .$$

$$S = b \cdot A$$

$$T\Delta S = \left(a \frac{m}{R^2}\right)\left(8\pi b R \Delta R\right) \qquad \boxed{ab = \frac{1}{8\pi}}$$

$$= 8\pi a b \, m \cdot \frac{\Delta R}{R} = 8\pi ab \cdot \Delta m = \Delta m .$$

$$\Box^2 \psi = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, g^{\mu\nu} \psi_{,\mu}\right)_{,\nu}$$

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

$$g = -r^4 \cdot \sin^2\theta \qquad \sqrt{-g} = r^2 \sin\theta$$

(3)

$$\frac{1}{\cancel{r^2 \sin\theta}} \partial_t \left( \cancel{r^2 \sin\theta} \, \frac{-1}{1-\frac{2m}{r}} \partial_t \psi \right)$$

$$+ \frac{1}{r^2 \cancel{\sin\theta}} \partial_r \left( r^2 \cancel{\sin\theta} \left(1-\frac{2m}{r}\right) \partial_r \psi \right)$$

$$+ \frac{1}{\cancel{r^2}\sin\theta} \partial_\theta \left( \cancel{r^2} \sin\theta \, \frac{1}{r^2} \partial_\theta \psi \right) = 0$$

$$+ \frac{1}{r^2 \sin\theta} \partial_\phi \left( r^2 \sin\theta \cdot \frac{1}{r^2 \sin^2\phi} \partial_\phi \psi \right)$$

$e^{i\omega t}$

$$-\frac{1}{1-\frac{2m}{r}} \frac{\partial^2 \psi}{\partial t^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \left(1-\frac{2m}{r}\right) \frac{\partial \psi}{\partial r} \right)$$

$$+ \frac{1}{r^2} \underbrace{\left[ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 \psi}{\partial \phi^2} \right)}_{Y_{\ell m}(\theta, \phi)} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( (r^2 - 2mr) \frac{\partial \psi}{\partial r} \right) + \frac{\omega^2}{1-\frac{2m}{r}} \psi - \frac{\ell(\ell+1)\psi}{r^2} = 0$$

$r \to \infty$ . $\frac{e^{\pm ikr}}{r} \cdot e^{-i\omega t} \, Y_{\ell m}(\theta, \phi)$

Green's function. periodic in imaginary time with period. ~~β~~ $8\pi M$

= Temperature!

$\langle f | e^{-iHt} | i \rangle = A_{fi}$ $\qquad Z = \mathrm{Tr}\, \langle n | e^{-\beta H} | n \rangle$

$$T = \beta^{-1} = \frac{1}{8\pi M} = \frac{1}{2\pi}\left(\frac{GM}{(2M)^2}\right) = \frac{\kappa}{2\pi}$$

$$T = \frac{\kappa}{2\pi} \qquad S = \frac{1}{4}A$$


: vacuum. $\langle a_- | 0 \rangle = 0$

→ ($\langle a_+ \rangle$). outgoing (density matrix) .. thermal.

$$T = \frac{1}{2\pi} \kappa$$ "Hawking radiation"

$$T_\odot = \frac{\hbar\kappa}{2\pi k_B c} = \frac{\hbar}{2\pi k_B c}\left(\frac{GM}{R^2}\right) = \frac{\hbar c^3}{8\pi G k_B M_\odot} = 0.06\ \mu K$$

$\hbar = 10^{-27}$ ~~erg~~ erg-s.
$= 6\cdot 10^{-16}$ eV-s $= 0.197$ GeV-fm

comoving → $(v\neq 0)$ in finite proper time

external observer → $e^{-t/M}$

Entropy loss ??

Thermal radiation. $\rho = \left(\frac{g\pi^2}{30}\right) T^4 = \frac{g\pi^2}{30}\,(kT)\left(\frac{kT}{\hbar c}\right)^3$

$$s = \frac{4}{3}\frac{\rho}{kT} = \frac{4}{3}\cdot\frac{g\pi^2}{30}\cdot\left(\frac{kT}{\hbar c}\right)^3$$

$s \simeq 10^{3}\ \mathrm{cm}^{-3}$ $\qquad S \simeq (10^{10}\ \mathrm{LY})^3 s = (10^{28}\ \mathrm{cm})^3 (10^{3}\ \mathrm{cm}^{-3})$

$\simeq 10^{87}$

BH$_\odot$. $A = 4\pi(3\ \mathrm{km})^2 = 10^{12}\ \mathrm{cm}^2$

$$S = \frac{1}{4}\frac{A}{\ell_{pl}^2} = \frac{1}{4}\left(\frac{m_{pl}c}{\hbar}\right)^2 A$$

$$= \frac{1}{4}\left(\frac{\hbar c}{G}\right)\left(\frac{c}{\hbar}\right)^2 = \frac{1}{4}\frac{c^3}{G\hbar}$$

$$S = \frac{1}{4}\cdot 4\pi\left(\frac{2\cdot 2\cdot 10^{33}\,g \cdot 6\cdot 10^{23}\ \mathrm{GeV}/g}{1.2\times 10^{19}\ \mathrm{GeV}}\right)^2 = 10^{76}$$

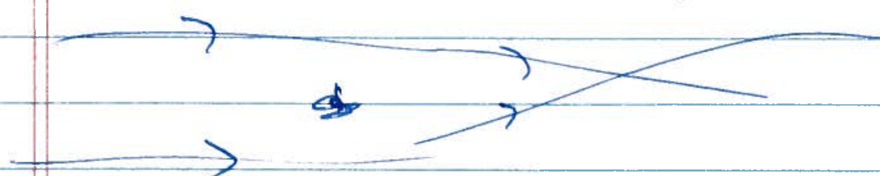
$S_{(3.6M)} = 10^{89}$ $\qquad S_{(10^9)} = 10^{94}$ $\times 10^{11}$

$n=3$ $\quad R_{\mu\nu}$: $\quad \frac{(3)(4)}{2} = 6 \; : \; \frac{n(n+1)}{2}$

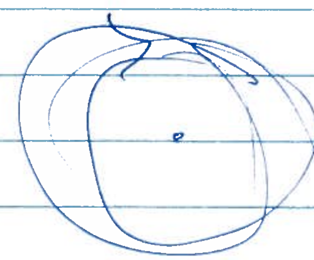
$R_{\alpha\beta\mu\nu}$: $\quad \frac{1}{12} n^2(n^2-1) = \frac{1}{12} \cdot 9 \cdot 8 = 6.$

$T_{\mu\nu}=0 \rightarrow R_{\mu\nu}=0 \rightarrow R^{\alpha}{}_{\beta\mu\nu}=0$

No local curvature effects.



$$\boxed{\Delta\Theta = 2\pi - (G\mu)}$$



$$\boxed{\Delta l = (2\pi - \delta) R}$$

"conic" singularity.