PHZ 7608 Spring 2017

Homework #1, Due Wednesday, February 1

1. A particle of mass m with energy ϵ and angular momentum ℓ is incident on a Schwarzschild black hole. For what values of ℓ/ϵ will the particle be captured by the hole? Write the capture cross section as a function of velocity. What are the Newtonian and strongly relativistic limits? (At least one of these should be familiar.)

2. Write the Schwarzschild or Reissner-Nordström metric as

$$ds^2 = -e^{2\alpha}\, dt^2 + e^{-2\alpha}\, dr^2 + r^2\, d\Omega^2.$$

(a) In the orthonormal basis $\tilde{\boldsymbol{\omega}}^{\hat{t}}$, $\tilde{\boldsymbol{\omega}}^{\hat{r}}$, $\tilde{\boldsymbol{\omega}}^{\hat{\theta}}$, $\tilde{\boldsymbol{\omega}}^{\hat{\phi}}$, compute the connection 1-forms $\tilde{\boldsymbol{\omega}}^{\hat{\mu}}{}_{\hat{\nu}}$.

(b) Compute the curvature 2-forms $\mathcal{R}^{\hat{\mu}}{}_{\hat{\nu}}$ and find the nonvanishing components of the Riemann tensor $R^{\mu\nu}{}_{\alpha\beta}$.

3. What are the values of the Schwarzschild t and r at the center of the Kruskal spacetime, $t' = 0$, $x' = 0$? What are the components of the Riemann tensor in t', x', θ, ϕ coordinates at this point?

4. The radial motion of a test particle of mass m and charge q with energy ϵ and angular momentum ℓ about a Reissner-Nordström black hole with mass M and charge Q is governed by

$$m^2\left(\frac{dr}{d\tau}\right)^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\left(m^2 + \frac{\ell^2}{r^2}\right) = \left(\epsilon - \frac{qQ}{r}\right)^2.$$

(a) Show that a test particle that crosses the horizon $r = r_+$ in the inward direction must have $\epsilon r_+ > qQ$.

(b) Show it is impossible to throw charged particles into a Reissner-Nordström black hole that starts with $Q^2 < M^2$ and thereby create one with $Q^2 > M^2$.

5. Express the proton mass m and charge e as lengths. Can a proton be a RN black hole?

6. Write the scalar wave equation $\Box^2\psi = 0$ in Schwarzschild and Reissner-Nordström spacetimes. Show that the equation separates, and has well known solutions for three of the four factors. Write the radial equation in terms of r and in terms of the tortoise coordinate r^* or its generalization in RN. How does the radial solution behave as $r \to \infty$? As r approaches the horizon?