PHZ 7608 Spring 2017

Homework #2, Due Wednesday, February 15

1. Express the neutron mass m and angular momentum per mass s/m as lengths. Can a neutron be a Kerr black hole?

2. What is the temperature of an extreme Reissner-Nordström black hole?

3. Compute the proper area of a surface of constant r and constant t in the Kerr geometry. Evaluate the general result on the horizon, $r = r_+$.

4. For a Kerr black hole ($Q = 0$), write p^t and p^ϕ in the equatorial plane $\theta = \frac{\pi}{2}$ in terms of the conserved quantities $p_t = -E$ and $p_\phi = L$. Determine an effective potential for radial motion in the equatorial plane. Try to identify possible circular orbits, and identify which are stable and unstable.

5. To leading order in a, find the rate of pericenter precession of an equatorial orbit about a Kerr black hole, assuming M/r is small. Is the difference between the precession rates for $a = 0$ and $a = 0.6\,M$ observable for the motion of the star S0-2 about Sgr A*?

6. Apply the concept of a *conformal Killing vector* (see the final exam for PHZ 6607 in the Fall) to the Robertson-Walker spacetime:

(a) Show by replacing time t with "conformal time" $\eta = \int dt/a(t)$ that the Robertson-Walker metric is conformally related to a time-independent metric that has an easily recognizable Killing vector.

(b) Use (a) and properties of a conformal Killing vector to obtain the relation between redshift and scale factor for photons.

7. A *Killing tensor* satisfies $K_{(\alpha\beta;\gamma)} = 0$.

(a) Show that if $K_{\alpha\beta}$ is a Killing tensor and $x^\mu(\lambda)$ is a geodesic with tangent $p^\mu = dx^\mu/d\lambda$, then $K_{\alpha\beta}\, p^\alpha p^\beta$ is a conserved quantity.

(b) Show that in Robertson-Walker spacetime, if $T^\mu = (1, 0, 0, 0)$ is the 4-velocity of a comoving observer, then $K_{\mu\nu} = a^2\,(g_{\mu\nu} + T_\mu T_\nu)$ is a conformal Killing vector.

(c) Let $x^\mu(\lambda)$ be a geodesic path with tangent p^μ. Use the conservation associated with the conformal Killing vector to obtain the evolution of the three-momentum $|\boldsymbol{p}|^2 = p \cdot p + (p_0)^2$ in a RW universe.