

$$1/9/2017 \quad r^t = r + 2m \ln(r - 2m).$$

Schwarzschild \rightarrow Eddington - Finkelstein

null

$$v = t + r^+$$

$$w = t - r^+$$

ingoing (advanced)
outgoing (retarded).

\rightarrow Kruskal - (Szekeres) (1960)

Schutz: u, v for ~~x, t~~ x', t'

Carroll: R, T for x', t'

$$v' = \exp\left(\frac{v}{4m}\right)$$

$$w' = \exp\left(-\frac{w}{4m}\right)$$

null

$$x' = \frac{1}{2}(v' + w')$$

$$x' = \frac{1}{2}(v' - w')$$

timelike

spacelike

(2)

Conformal diagram (Penrose).

$$\text{Flat space: } -dt^2 + dx^2 = -d\bar{w} dw.$$

$$v = t+x \quad d\bar{w} dw = (dt+dx)(dt-dx) \\ w = t-x \quad = \underline{dt^2 - dx^2}$$



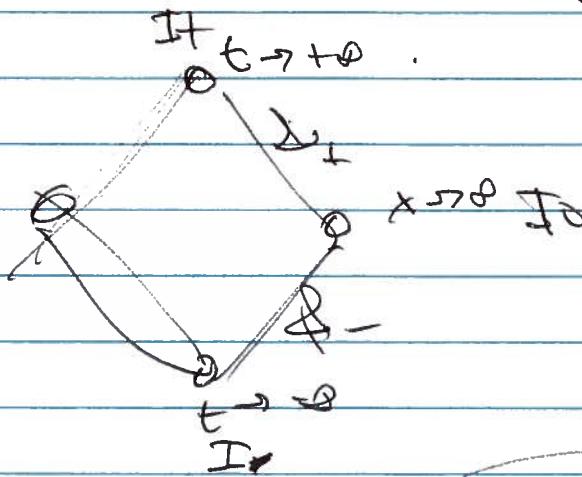
$$V = \arctan v$$

$$W = \arctan w$$

$$v = \tan V$$

$$w = \tan W$$

$$dv dw = \left(\frac{\partial V}{1+v^2}\right)\left(\frac{\partial W}{1+w^2}\right) = \frac{dv dw}{(1+v^2)(1+w^2)^2}.$$



Schwarzschild.

$$V' = \arctan v'$$

$$W' = \arctan w'$$

3

In the Fall : charged b.f.

$$\tilde{\tau}^0 = \rho = \frac{1}{8\pi} (\tilde{\epsilon}^2 + \tilde{B}^2)$$

$$\tilde{A}^m = (0, 0, 0, 0)$$

$$\nabla^2 \tilde{A}^m = 4\pi \tilde{\rho} \quad \Rightarrow \quad \nabla \cdot \tilde{E} = \rho \quad \Rightarrow \quad \tilde{E} = \frac{1}{\sqrt{-g}} \hat{e}_t$$



$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) dt^2 + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$u_t = -\tilde{E} = \text{constant}$$

$$u_\varphi = \tilde{L} = \text{constant}$$

$$\left(\frac{dr}{dt}\right)^2 + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) \left(1 + \frac{r^2}{\tilde{L}^2}\right) = \frac{r^2}{\tilde{E}}$$

\rightarrow no em. force!

(4)

Horizons: $r^2 = 2mr + e^2 \Rightarrow$

$$\hookrightarrow r_{\pm} = m \pm \sqrt{m^2 - e^2}$$

$$e^2 < m^2$$

\rightarrow Penrose.

Charged particle:

$$\frac{du^{\mu}}{d\tau} = m \frac{du^{\mu}}{dx^i} = q_e F^{\mu\nu} u_{\nu}$$

$$\hookrightarrow m D_u u_{\mu} = m u^{\beta} (u_{\alpha\beta} - P_{\alpha\beta}^{\mu} u^{\mu})$$

$$= m \frac{du^{\mu}}{dx^i} - m P_{\alpha\beta}^{\mu} u^{\alpha} u^{\beta}$$

$$\hookrightarrow \frac{du^{\mu}}{d\tau} = \frac{1}{2} g_{\alpha\beta} m u^{\alpha} u^{\beta} + \frac{q_e}{m} F_{\mu\nu} u^{\nu}$$

" $F = ma$ " gravitational + E.m.

(5)

$$\text{Still true: } \frac{\partial u_t}{\partial r} = 0 \quad u_t = \tilde{L} = \text{constant}$$

$$\frac{\partial u_t}{\partial r} = \frac{1}{2} g_{tt} u u_r + \frac{g}{m} \tilde{F}_{tr} u^r = \frac{g}{m} \tilde{F}_{tr} u^r$$

$$= \frac{g}{m} (-E) \frac{\partial r}{\partial r} = + \frac{g}{m} \left(\frac{\partial r}{\partial r} \right) \frac{\partial r}{\partial r}$$

$$\begin{pmatrix} 0 & E^{00} \\ E^{00} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \frac{\partial}{\partial r} \left(u_t - \frac{g}{m} \right) = 0$$

$$\Rightarrow u_t - \frac{g}{m} = -\tilde{E}$$

$$u_t = -\tilde{E} + \frac{g}{m}$$

$$\underbrace{u \cdot u = -1}$$

$$\left(\frac{\partial r}{\partial r} \right)^2 + \left(1 - \frac{2m}{r} + \frac{g^2}{r^2} \right) \left(1 + \frac{\tilde{L}^2}{r^2} \right) = \left(\tilde{E} - \frac{g}{m} \right)^2$$

$$\text{non-rel.} \quad 1 + \frac{1}{r} - \frac{2m}{r} + \frac{\tilde{L}^2}{r^2} = \left(\tilde{E} - \frac{g}{m} \right)^2$$

$$1 + \frac{1}{2} \frac{1}{r} - \frac{m}{r} + \frac{\tilde{L}^2}{2r^2} = \tilde{E} - \frac{g}{m}$$

$$\frac{1}{2} \frac{m}{r} - \frac{(G)m}{r} + \frac{g}{r} + \frac{\tilde{L}^2}{2mr^2} = \tilde{E} - \frac{g}{m}$$