

$$\boxed{3/3 \text{ (20c)}} \quad \text{Scalar field dynamics: } S = \int d^3x \left(-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right)$$

$$\rightarrow \ddot{\phi} + \frac{3}{a^2} \dot{\phi} - \frac{1}{a^2} \nabla^2 \phi = - \frac{\partial V}{\partial \phi}$$

$$\left(\frac{\dot{\phi}}{a} \right)^2 = \frac{8\pi G}{3} \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} a^2 (\nabla_\mu \phi)^2 + V \right)$$

"Slow-roll" inflation

$$\dot{\phi} = \frac{(-V')}{3 \left(\frac{8\pi G}{3} V \right)^{1/2}} = \frac{(-V')}{3 \left(\frac{1}{3} \bar{m}_{pl}^2 V \right)^{1/2}}$$

$$\frac{\dot{\phi}^2}{V} = \frac{\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{V'^2}{V} \bar{m}_{pl}^2}{V} = \frac{1}{3} \left(\frac{1}{2} \bar{m}_{pl}^2 \frac{V'^2}{V^2} \right) \uparrow \epsilon$$

$$\ddot{\phi} = \frac{d\dot{\phi}}{dt} = \frac{d\dot{\phi}}{d\phi} \cdot \frac{d\phi}{dt} = \frac{d}{d\phi} \left(\frac{1}{2} \dot{\phi}^2 \right)$$

$$= \dot{\phi} \left(\frac{1}{2} \bar{m}_{pl}^2 \frac{V'^2}{V} \right) = \frac{1}{3} \cancel{\dot{\phi} V'} \cdot \bar{m}_{pl}^2 \left(\frac{V''}{V} - \frac{V'^2}{2V^2} \right)$$

$$= \frac{V'}{3} \left(\bar{m}_{pl}^2 \frac{V''}{V} - \frac{1}{2} \bar{m}_{pl}^2 \frac{V'^2}{V^2} \right)$$

(2)

$$\frac{dV}{dt} = \frac{1}{3} \left(\frac{\text{temp}^2 V''}{V} - \varepsilon \right)$$

↑
→

$$\left(\varepsilon = \frac{1}{2} \frac{\text{temp}^2}{V} \left(\frac{dV}{dt} \right)^2 \right)$$

$$\left(\eta = \frac{\text{temp}^2}{V} \frac{dV}{dt} \right)$$

ε_{ext} , η_{ext} \rightarrow $\rho \approx \text{constant}$

$$\underbrace{a \approx e^{xt}}_{\text{for a while}}.$$

Eventually: must end.

dynamics + interaction \rightarrow "heating"

(3)

New minimum: $V \approx \frac{1}{2}m^2\dot{\phi}^2$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -m^2\phi$$

$$m^2 \gg \left(\frac{\dot{a}}{a}\right)^2$$

oscillating: $\phi = A \cos \omega t$

$$A(t), \omega(t)$$

~~Slowly varying~~
~~A $\ll m$~~ ~~$\omega \ll m$~~

$$\dot{\phi} = \dot{A} \cos \omega t + A \dot{\omega} \sin \omega t$$

$$\dot{A} \approx m$$

$$\dot{\omega} \ll m$$

$$\ddot{\phi} = \ddot{A} \cos \omega t - 2\dot{A}\dot{\omega} \sin \omega t - A \ddot{\omega} \cos \omega t - A \dot{\omega}^2 \sin \omega t =$$

$$\omega_0 \left(\ddot{A} - \frac{A \dot{\omega}^2}{\omega^2} \right) - \sin \theta \left(2\dot{A}\dot{\omega} + A \dot{\omega}^2 \right)$$

$$+ \frac{3\dot{a}}{a} \left(\dot{A} \cos \omega t - A \dot{\omega} \sin \omega t \right) = - \frac{m^2 \cdot A \cdot \cos \omega t}{\omega^2}$$

$$O(\omega^2)$$

$$\overbrace{\theta}^2 \approx \overbrace{m^2}^2$$

$$O(\omega): -2\dot{A}\dot{\omega} - \frac{3\dot{a}}{a} A \dot{\omega} = 0$$

$$\dot{A} + \frac{3\dot{a}}{2\dot{a}} A = 0 \quad | A \propto a^{-3/2}$$

(4)

$$\text{Q(1)} \quad \cos\theta (A - A\dot{\phi}_1^2) - \sin\theta (2A\dot{\phi}_1 + A\ddot{\phi}_1) \\ + 3\frac{\dot{a}}{a} (A\cos\theta - A(\dot{\phi}_1^2 + \dot{\phi}_1^2)) = -m^2 A \cos\theta$$

$$\text{Q(2)} \quad \omega_3 (A - A(m^2 + \dot{\phi}_1^2)) - \sin\theta (2A m (\dot{\phi}_1 + \dot{\phi}_1) - A\ddot{\phi}_1) \\ + 3\frac{\dot{a}}{a} (A\cos\theta - A(\dot{\phi}_1^2 + \dot{\phi}_1^2) \sin\theta) = -m^2 A \cos\theta$$

$\sqrt{a \approx t^{2/3}}$

$$P = \frac{1}{2}\dot{\epsilon}^2 + \frac{1}{2}m^2\dot{\phi}^2 = \frac{1}{2}A^2(-m\sin\theta)^2 + \frac{1}{2}m^2A^2\omega_3^2$$

$$= \frac{1}{2}A^2m^2(\sin^2\theta + \cos^2\theta) = \frac{1}{2}A^2m^2$$

$\checkmark P \propto \dot{a}^{-3}(t)$

"matter dominated",
(axion).

$$\dot{\phi}^2 \sim p \sim \dot{a}^{-\frac{q}{2}} \quad | \quad \dot{\phi} \propto \dot{a}^{-\frac{q}{2}(2)} \quad \alpha t^{\frac{2-q}{2}} = (t)$$

$$(\frac{\dot{a}}{a})^2 = \frac{p^2}{t^2} \propto \dot{a}^{-\frac{q}{2}} \quad | \quad a \propto t^{2/q}$$

$$\dot{\phi} = [p + \dot{p}]^{\frac{1}{2}}$$

$$(\frac{\partial \dot{\phi}}{\partial a}) = \frac{1}{a} = \frac{\dot{p}}{a(\frac{\dot{a}}{a})} =$$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3} p$$

$$\begin{aligned} f &= \frac{1}{2} \dot{\phi}^2 + V \\ p &= \frac{1}{2} \dot{\phi}^2 - V \end{aligned} \quad \left\{ \quad \dot{\phi}^2 = f + p = (1+w)\rho$$

$$\left(\frac{\dot{\phi}}{a}\right)^2 = \frac{8\pi}{3} G\rho, \quad \dot{\phi} \propto \left(\frac{a}{r}\right)$$

$$\frac{\dot{\phi}^2}{(a)^2} = \frac{(1+w)\rho}{\frac{8\pi}{3} G w \rho r^2} = \frac{3(1+w) w \rho r^2}{8\pi}$$

$= f \left(\frac{w\rho r^2}{8\pi}\right)$.

$$\frac{\partial f}{\partial a} = \frac{\dot{\phi}}{a} = \frac{\sqrt{f \frac{w\rho r^2}{8\pi}}}{a}$$

$$\frac{\partial \phi}{\partial (\ln a)} = \sqrt{f \frac{w\rho r^2}{8\pi}}$$

$$f = (\cdot) \cdot \ln a$$

$$a = \exp \left(\underbrace{\sqrt{\frac{8\pi}{3} w \rho r^2 f}}_{\text{constant}} \cdot (\phi - \phi_0) \right)$$