

4/3/2017

$$ds^2 = S(\bar{t}) \left[ - (1 + 2A^{(0)}(\bar{x})) dt^2 + 2 B^{(0)}(\bar{x}) dt dx^i + (1 + 2H^{(0)}(\bar{x})) h_{ij} dx^i dx^j + 2H_T^{(0)}(\bar{x}) \underbrace{Q_j^{(0)}}_{+ (0) \omega_j} dx^i dx^j \right]$$

Gauge transformation scalar

$$\bar{t} = t + T^{(0)}(\bar{x})$$

$$\bar{x}^i = x^i + L^{(0)}(\bar{x}) Q^{(0)i}(\bar{x})$$

$$g_{\alpha\beta} = \frac{\partial x^k}{\partial \bar{x}^\alpha} \frac{\partial x^l}{\partial \bar{x}^\beta} \tilde{g}_{kl}$$

$$S(\bar{t}) \approx S(t) \left[ 1 + (\dot{S}/S) T Q^{(0)} \right]$$

$$\tilde{h}_{ij} \approx h_{ij} + L^{(0)} Q^{(0)k} \frac{\partial}{\partial x^k} h_{ij}$$

metric derivatives  
combine with  $\partial$   
to give  $\partial Q$

$$\begin{aligned} \tilde{A} &= A - \dot{T} - (\dot{S}/S) T \\ \tilde{B}^{(0)} &= B^{(0)} + L^{(0)} + kT \\ \tilde{H}_L &= H_L - (k/3) L^{(0)} - (\dot{S}/S) T \\ \tilde{H}_T^{(0)} &= H_T^{(0)} + k L^{(0)} \end{aligned}$$

$$\frac{\partial^{(0)l} h_{li} + 2 \partial^{(0)} \rightarrow \partial}{\partial x^k} = \frac{1}{k} \partial^{(0)} h_{li}$$

$$\vec{v}^{(0)} \cdot \vec{\Theta}^{(0)} = \frac{d\vec{x}^{(0)}}{d\tau} \approx \frac{d\vec{x}^{(0)}}{dt} + \dot{L}^{(0)} \cdot \vec{\Theta}^{(0)}$$

$$\rightarrow \vec{v}^{(0)} = \vec{v}^{(0)} + \dot{L}^{(0)}$$

$$\vec{E}(\vec{r}) = \epsilon_0 \vec{E} [1 + \vec{\delta} \cdot \vec{\Theta}^{(0)}]$$

$$\approx \epsilon_0(\vec{r}) [1 + (\vec{\delta} + T \dot{\epsilon}_0 / \epsilon_0) \cdot \vec{\Theta}^{(0)}]$$

$$\rightarrow \vec{\delta} = \delta + 3(1+w)(\dot{S}/S)T \quad \left[ \dot{\epsilon}_0 = (\epsilon_0 + P_0) \left( \frac{\dot{S}}{S} \right) \right]$$

(-3S/S)

$$\vec{\pi}_L = \vec{\pi}_L - T \dot{P}_0 / P_0 = \vec{\pi}_L + 3(1+w) \frac{c^2 \dot{S}}{w S} T$$

$\vec{\pi}_T^{(0)}$  unchanged  $\Phi$

gauge conditions: (Some examples)

"synchronous"  $A = B^{(0)} = 0$

"longitudinal"  $H_T^{(0)} = B^{(0)} = 0$

"comoving, time-orthogonal"  $B^{(0)} = v^{(0)} = 0$

may or may not specify completely

(B)

Only gauge-invariant quantities have inherent physical meaning.

Metric: 4 functions  $A, B, H_L, H_T$ ,  
gauge 2 ~~gauge~~  $T, L$

$$\Phi_A = A + \frac{1}{k} \dot{B}^{(0)} + \frac{1}{kS} \dot{B}^{(0)} - \frac{1}{k^2} \left( \ddot{H}_T^{(0)} + \frac{\dot{S}}{S} \dot{H}_T^{(0)} \right)$$

$$\Phi_H = H_L + \frac{1}{3} \dot{H}_T^{(0)} + \frac{1}{kS} \dot{B}^{(0)} - \frac{1}{k^2} \frac{\dot{S}}{S} \dot{H}_T^{(0)}$$

$$V_S^{(0)} = V^{(0)} - \frac{1}{k} \dot{H}_T^{(0)}$$

↔ matter shear

$$\sigma_{\alpha\beta} = \frac{1}{2} P_{\alpha}^{\mu} (u_{\mu;j} + u_{j;\mu}) P_{\beta}^{\lambda} - \frac{1}{3} P_{\alpha\beta} u^{k}_{;k}$$

$$\sigma^2 = \frac{1}{2} \sigma_{\alpha\beta} \sigma^{\alpha\beta} = \left( \frac{k}{S} V_S^{(0)} \right)^2 \left( \frac{1}{2} \theta^{\omega\alpha\beta} \theta_{\alpha\beta}^{\omega} \right)$$

(8) = want (gauge invariant) →  $\delta$   
"inside horizon".  $\dot{S}/S \ll 1$  ( $H \ll k/k_H$ )

Two choices  $E_m = \delta + 3(1+w) \frac{1}{kS} (V^{(0)} - B^{(0)})$

(4)

$\Sigma_m = \delta$  whenever  $v^{(0)} = \beta^{(0)}$   
 $\leftrightarrow$  matter worldlines orthogonal to  $(t = \text{constant})$

"natural choice from the point of view of the matter"

$$\Sigma_g = \delta - 3(\kappa\omega) \frac{1}{k} \frac{\delta}{\delta} \left( \beta^{(0)} - \frac{1}{k} \dot{H}_T^{(0)} \right)$$

$\Sigma_g = \delta$  when  $\beta^{(0)} = \frac{1}{k} \dot{H}_T^{(0)}$   $\leftrightarrow$   $\sigma^2 = 0$

"As close as possible to a "Newtonian" time slicing"

any linear combination is also invariant

$\Sigma_m - \Sigma_g = 3(\kappa\omega) \frac{1}{k} \frac{\delta}{\delta} v_s^{(0)}$  small  $\Delta \ll \tau$   
inside horizon

Typically:  $\Sigma_m \propto (kc)^3$   $\Sigma_g$  constant

for perturbations regular as  $S \rightarrow 20$ .

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When  $B^{(0)} = \frac{1}{k} H_T^{(0)}$

↔  $\tau = \text{constant}$  surfaces have zero-shear waves

$\Phi_A = A$

$\Phi_H = H_L + \frac{1}{3} H_T^{(0)}$

↔  $\Phi_A =$  "lapse" between  $\tau, \tau + \delta\tau$

$R_{\text{shear}} = [6k + 4(k^2 - 3k) \Phi_H \delta^{(0)}] S^{-2}$

↔  $\Phi_H =$  "curvature perturbation"

longitudinal gauge.

$\Phi_A = A$

$\Phi_H = H_L$

$v_s = v$

$\epsilon g = \delta$

$2w = \delta$

$v^{(0)} - B^{(0)} = 0$

$H_T^{(0)} = 0$

(6)

Vector perturbations  $\propto \theta^{(1)}$

$$\tilde{x}^i = x^i + L^{(1)} \theta^{(1)i} \quad (\propto T^{(1)})$$

$$\tilde{B}^{(1)} = B^{(1)} + L^{(1)}$$

$$\tilde{H}_T^{(1)} = H_T^{(1)} + k L^{(1)}$$

$$\tilde{v}^{(1)} = v^{(1)} + \dot{L}^{(1)}$$

metric  $\Psi = B^{(1)} - \frac{1}{k} \dot{H}_T^{(1)}$

$\frac{k\Psi}{S} =$  ~~shear~~ of  $(\bar{c} = \text{constant})$  normaly.

velocity two choices:  $v_s^{(1)} = v^{(1)} - \frac{1}{k} \dot{H}_T^{(1)}$   
(shear).

vorticity  $w_{AB} = \frac{1}{2} P_A^k (u_{k;j} - u_{j;k}) P_B^j$

$$w_{ij} = S (v^{(1)} - B^{(1)}) (\theta^{(1)}_{i;j} - \theta^{(1)}_{j;i})$$

$\propto k w_{ij}$

$$\omega^2 = \frac{1}{2} w_{ij} w^{ij} = \frac{k^2 (v^{(1)} - B^{(1)})^2}{S^2} \left( \frac{1}{2} w_{ij} w^{ij} \right)$$

(measurable).

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$$v_c = v^{(c)} - B^{(c)} = v_s^{(c)} - \dot{\Phi}$$

↑ velocity relative to (normal) to  $(\kappa = \text{constant})$ .  
circulatory.

tensor perturbations:  $h_T^{(2)}$ ,  $\pi_T^{(2)}$   
are gauge invariant.

Einstein:  $\delta G^d_B = \delta R^d_B - \frac{1}{2} \delta^d_B \delta R$ .

Scalar  $\delta G^0_0 = \frac{3}{k^2} \delta ( \delta G^0_j )^{,j}$

$$= -2 \frac{(k^2 - 3k)}{k^2} \Phi_H \partial^{(0)}$$

$$\delta G^i_j - \frac{1}{3} \delta^i_j \delta G^k_k = -\frac{k}{k^2} (\Phi_A + \Phi_H) \partial^{(0)} \delta^i_j$$

$$\frac{2(k^2 - 3k)}{k^2} \Phi_H = \epsilon_0 \epsilon_m$$

$$-\frac{k^2}{k^2} (\Phi_A + \Phi_H) = P_0 \pi_T^0$$

perfect fluid. ( $\pi_T = 0$ )

$$\Phi_H = -\Phi_A$$