

The invariant measure for scattering matrices with block symmetries

Víctor A Gopar[†], Moisés Martínez[†], Pier A Mello[†] and Harold U Baranger[‡]

[†] Instituto de Física, Universidad Nacional Autónoma de México, 01000 México DF, Mexico

[‡] AT&T Bell Laboratories, 600 Mountain Avenue 1D-230, Murray Hill, NJ 07974-0636, USA

Received 31 July 1995

Abstract. We find the invariant measure for two new types of S matrices relevant for chaotic scattering from a cavity in a waveguide. The S matrices considered can be written as a 2×2 matrix of blocks, each of rank N , in which the two diagonal blocks are identical and the two off-diagonal blocks are identical. The S matrices are unitary; in addition, they may be symmetric because of time-reversal symmetry. The invariant measure, with and without the condition of symmetry, is given explicitly in terms of the invariant measures for the well known circular unitary and orthogonal ensembles. Some implications are drawn for the resulting statistical distribution of the transmission coefficient through a chaotic cavity.

1. Introduction

A wave-scattering problem can, very generally, be described by its scattering matrix S (Newton 1966). In a stationary problem, S relates the outgoing-wave to the ingoing-wave amplitudes. The condition of flux conservation implies unitarity of S ,

$$SS^\dagger = I. \quad (1)$$

If, in addition, the problem is invariant under the operation of time reversal, S is symmetric,

$$S = S^T. \quad (2)$$

If one desires a statistical description of the scattering, the problem of assigning ‘equal *a priori* probabilities’ in the space of scattering matrices S (Hua 1963, Dyson 1962) may be relevant, and, in fact, has been shown to be important for the description of chaotic scattering (Mello *et al* 1985, Blümel and Smilansky 1988, 1989, 1990, Lewenkopf and Weidenmüller 1991, Jalabert *et al* 1994, Baranger and Mello 1994, 1995). The assignment is done through the notion of the *invariant measure*: the measure $d\mu^{(\beta)}(S)$ which is invariant under the symmetry operations for the universality class, labelled by β , in question. Explicitly, $d\mu^{(\beta)}(S) = d\mu^{(\beta)}(U_0 S V_0)$ where U_0, V_0 are arbitrary fixed unitary matrices in the case of unitary S matrices (the circular unitary ensemble ($\beta = 2$)), with the restriction $V_0 = U_0^T$ in the case of unitary symmetric S matrices (the circular orthogonal ensemble ($\beta = 1$)) (Hua 1963, Dyson 1962, for a review see Mehta 1991).

As an example, consider single-electron scattering by a ballistic quantum dot connected to the outside by two leads, which play the role of waveguides, each with N transverse modes or channels (for a review see Beenakker and van Houten 1991). The S matrix is then $2N$ -dimensional and has the structure

$$S = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix} \quad (3)$$

where r, r' are the $N \times N$ reflection matrices (for incidence from either lead) and t, t' the corresponding transmission matrices. Of great physical relevance is the total transmission coefficient,

$$T = \text{tr}(tt^\dagger) \quad (4)$$

which is proportional to the conductance G of the cavity, $G = (2e^2/h)T$. The invariant measure for the S matrix implies a probability distribution $w(T)$ for T which has been calculated in a number of cases by Jalabert *et al* (1994) and Baranger and Mello (1994).

In the present article we study S matrices of the form (3), with the restriction $r = r', t = t'$; i.e.

$$S = \begin{bmatrix} r & t \\ t & r \end{bmatrix}. \quad (5)$$

S matrices with this structure are physically relevant because it is possible, in principle, to study electron transport through chaotic cavities with point spatial symmetries. Consider a two-dimensional cavity connected to two parallel waveguides. With the condition of unitarity alone, the S matrices (5) are appropriate for a system with inversion symmetry with respect to a point but no time-reversal symmetry (Baranger and Mello 1996). With the additional condition (2), equation (5) describes a system which is time-reversal invariant and has either inversion symmetry or symmetry with respect to a line perpendicular to the waveguides (Baranger and Mello 1996).

The invariant measure for matrices of the form (5), with and without the condition of symmetry (2), is obtained in section 2. A number of important expectation values are obtained, for an arbitrary number of channels N , in section 3. The probability density $w(T)$ arising from the invariant measure is obtained in section 4.1 for $N = 1$; $w(T)$ for $N = 2$ is found in section 4.2 for the S matrices (5) with the condition of symmetry (2) and in section 4.3 without the symmetry condition.

2. The invariant measure for S matrices with $r = r', t = t'$

All of the $2N$ -dimensional S matrices with the structure (5) can be simultaneously brought to block-diagonal form by using the rotation matrix

$$R_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1_N & 1_N \\ -1_N & 1_N \end{bmatrix} \quad (6)$$

where 1_N is the N -dimensional unit matrix:

$$S' = R_0 S R_0^T = \begin{bmatrix} s^{(+)} & 0 \\ 0 & s^{(-)} \end{bmatrix}. \quad (7)$$

Since S is unitary, so are S' and the two $N \times N$ matrices $s^{(\pm)} = r \pm t$. Clearly, two arbitrary unitary matrices $s^{(\pm)}$ can generate the most general unitary S matrix with the structure (5) by taking

$$r = \frac{1}{2}[s^{(+)} + s^{(-)}] \quad t = \frac{1}{2}[s^{(+)} - s^{(-)}]. \quad (8)$$

If, in addition, S is symmetric, so are r, t and $s^{(\pm)}$. The total number of independent parameters of unitary matrices with the structure (5) is thus $2N^2$ without and $N(N + 1)$ with the symmetry requirement.

The most general automorphism of unitary matrices with the structure (5) is generated by the transformation

$$\tilde{s}^{(\pm)} = u_0^{(\pm)} s^{(\pm)} v_0^{(\pm)} \quad (9)$$

where $u_0^{(\pm)}, v_0^{(\pm)}$ are arbitrary but fixed $N \times N$ unitary matrices. Correspondingly, the original S is transformed into

$$\tilde{S} = U_0 S V_0 \tag{10}$$

with the $2N \times 2N$ unitary matrix U_0 given by

$$U_0 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \quad \alpha = \frac{1}{2}[u_0^{(+)} + u_0^{(-)}] \quad \beta = \frac{1}{2}[u_0^{(+)} - u_0^{(-)}] \tag{11}$$

and a similar expression holds for V_0 with $v_0^{(\pm)}$ replacing $u_0^{(\pm)}$. In this automorphism, there is the restriction $v_0^{(\pm)} = [u_0^{(\pm)}]^T$ if the S matrices are also symmetric.

Denoting by $d\hat{\mu}^{(\beta)}(S)$ the invariant measure for S matrices of the form (5), with (without) the condition of symmetry for $\beta = 1$ (2), we thus have

$$d\hat{\mu}^{(\beta)}(S) = d\mu^{(\beta)}(s^{(+)}) d\mu^{(\beta)}(s^{(-)}). \tag{12}$$

3. Expectation values

A number of expectation values have been calculated for the circular orthogonal ensemble by Mello and Seligman (1980) and for the circular unitary ensemble by Mello (1990). These results will be employed here to evaluate various expectation values of physical interest for the invariant measure (12).

We use the notation

$$Q_{c_1\gamma_1, \dots, c_m\gamma_m}^{a_1\alpha_1, \dots, a_l\alpha_l}(\beta) \equiv \langle [s_{c_1\gamma_1} \dots s_{c_m\gamma_m}] [s_{a_1\alpha_1} \dots s_{a_l\alpha_l}]^* \rangle^{(\beta)} \tag{13}$$

to indicate an expectation value for the circular ensemble β for N -dimensional unitary s matrices. One can show that $Q = 0$ unless $m = l$. A simple application of this result is the expectation value

$$\left. \begin{matrix} \langle r_{ab} \rangle^{(\beta)} \\ \langle t_{ab} \rangle^{(\beta)} \end{matrix} \right\} = \frac{1}{2} [\langle s_{ab}^{(+)} \rangle^{(\beta)} \pm \langle s_{ab}^{(-)} \rangle^{(\beta)}] = 0 \tag{14}$$

for $a, b = 1, \dots, N$.

Using the statistical independence of $s^{(+)}$ and $s^{(-)}$, equation (12), one concludes immediately that

$$\langle r_{ab} r_{cd}^* \rangle^{(\beta)} = \langle t_{ab} t_{cd}^* \rangle^{(\beta)} = \frac{1}{2} Q_{ab}^{cd}(\beta) \quad \langle r_{ab} t_{cd}^* \rangle^{(\beta)} = 0. \tag{15}$$

In particular, the average of individual reflection and transmission coefficients is given by

$$\langle |r_{ab}|^2 \rangle^{(\beta)} = \langle |t_{ab}|^2 \rangle^{(\beta)} = \frac{1}{2} Q_{ab}^{ab}(\beta) \tag{16}$$

and the average of the total transmission coefficient of equation (4) is

$$\langle T \rangle^{(\beta)} = \frac{1}{2} \sum_{ab} Q_{ab}^{ab}(\beta). \tag{17}$$

The cross second moment of individual transmission coefficients is

$$\langle |t_{ab}|^2 |t_{cd}|^2 \rangle^{(\beta)} = \frac{1}{8} \{ Q_{ab,cd}^{ab,cd}(\beta) + Q_{ab}^{ab}(\beta) Q_{cd}^{cd}(\beta) + [Q_{ab}^{cd}(\beta)]^2 \}. \tag{18}$$

Summing over indices, we find $\langle T^2 \rangle$ and obtain the variance of T :

$$\text{var } T^{(\beta)} \equiv \langle (T - \langle T \rangle)^2 \rangle^{(\beta)} = \frac{1}{8} \sum_{abcd} [Q_{ab}^{cd}(\beta)]^2. \tag{19}$$

In order to calculate definite values for $\langle T \rangle$ and $\text{var } T$, recall that for $\beta = 1$, Mello and Seligman (1980) find

$$Q_{c\gamma}^{a\alpha}(\beta = 1) = \frac{\delta_c^a \delta_\gamma^\alpha + \delta_c^\alpha \delta_\gamma^a}{N + 1} \quad (20)$$

while for $\beta = 2$, Mello (1990) shows

$$Q_{c\gamma}^{a\alpha}(\beta = 2) = \frac{\delta_c^a \delta_\gamma^\alpha}{N}. \quad (21)$$

Using these expressions in (17) and (19), we obtain

$$\langle T \rangle^{(\beta)} = \frac{N}{2} \quad (22a)$$

$$\text{var } T^{(\beta=1)} = \frac{N}{4(N + 1)} \quad (22b)$$

$$\text{var } T^{(\beta=2)} = \frac{1}{8}. \quad (22c)$$

The result for $\langle T \rangle$ is expected since reflection and transmission are statistically equivalent, equations (8) and (12). As $N \rightarrow \infty$, $\text{var } T$ tends to the universal result $\frac{1}{4}$ for $\beta = 1$, while for $\beta = 2$, $\text{var } T$ is completely independent of N .

4. The probability distribution of T

4.1. The case $N = 1$

Writing

$$s^{(\pm)} = \exp[i\theta^{(\pm)}] \quad (23)$$

we have from (8)

$$T = \frac{1}{2}[1 - \cos(\theta^{(+)} - \theta^{(-)})] \quad (24)$$

for both $\beta = 1$ and $\beta = 2$. The probability distribution $w(T)$ of T is an average over the angles and can be written as

$$w(T) = \left\langle \delta \left[T - \frac{1 - \cos(\theta^{(+)} - \theta^{(-)})}{2} \right] \right\rangle_{\theta^{(+)}, \theta^{(-)}}. \quad (25)$$

Equation (12) implies that $\theta^{(+)}$ and $\theta^{(-)}$ are statistically independent and uniformly distributed in $(0, 2\pi)$, yielding

$$w(T) = \frac{1}{\pi \sqrt{T(1-T)}}. \quad (26)$$

4.2. The case $N = 2$, $\beta = 1$

The most general 2×2 unitary symmetric matrices $s^{(\pm)}$ can be written as

$$s^{(\pm)} = \begin{bmatrix} -\sqrt{\rho^\pm} e^{2i\alpha^\pm} & \sqrt{\tau^\pm} e^{i(\alpha^\pm + \gamma^\pm)} \\ \sqrt{\tau^\pm} e^{i(\alpha^\pm + \gamma^\pm)} & \sqrt{\rho^\pm} e^{2i\gamma^\pm} \end{bmatrix} \quad (27)$$

where α^\pm and γ^\pm are defined in $(0, 2\pi)$, $0 \leq \rho^\pm \leq 1$, $0 \leq \tau^\pm \leq 1$, and $\rho^\pm + \tau^\pm = 1$. The invariant measure for $s^{(\pm)}$ is (Jalabert et al 1994, Baranger and Mello 1994)

$$d\mu^{(1)}(s^{(\pm)}) \propto \frac{d\tau^\pm}{\sqrt{\tau^\pm}} d\alpha^\pm d\gamma^\pm. \quad (28)$$

In terms of these variables, the probability density $w(T')$ for $T' = 1 - T \in (-1, 1)$ is

$$w(T') \propto \int \frac{d\tau^+ d\tau^-}{\sqrt{\tau^+ \tau^-}} d\alpha^+ d\gamma^+ d\alpha^- d\gamma^- \delta \left\{ T' - \frac{1}{2} \sqrt{\rho^+ \rho^-} [\cos 2(\alpha^+ - \alpha^-) + \cos 2(\gamma^+ - \gamma^-)] - \sqrt{\tau^+ \tau^-} \cos(\alpha^+ - \alpha^- + \gamma^+ - \gamma^-) \right\}. \tag{29}$$

The integrals over α^- and γ^- are trivial. Defining $2\alpha^+ = \phi + \psi$ and $2\gamma^+ = \phi - \psi$, we have

$$w(T') \propto \int \delta \left[T' - \left(\sqrt{\tau^+ \tau^-} + \sqrt{\rho^+ \rho^-} \cos \psi \right) \cos \phi \right] d\sqrt{\tau^+} d\sqrt{\tau^-} d\phi d\psi. \tag{30}$$

The integral over ϕ gives

$$w(T') \propto \int_0^1 dz \int_0^1 dz' \int_0^{2\pi} d\psi \frac{u(X^2 - T'^2)}{\sqrt{X^2 - T'^2}} \tag{31}$$

where $u(X^2 - T'^2)$ is a step function, $z = \sqrt{\tau^+}$, $z' = \sqrt{\tau^-}$, and

$$X = zz' + \sqrt{(1 - z^2)(1 - z'^2)} \cos \psi. \tag{32}$$

Notice that changing ψ to $\psi + \pi$, which does not alter the integral in (30), is equivalent to changing zz' to $-zz'$ in X^2 . In (31) we can thus extend the range of integration of the variables z and z' to the interval $(-1, 1)$. Furthermore, we can write, in (31), $\psi = \varphi - \varphi'$ and integrate φ and φ' separately from 0 to 2π without altering the answer (up to a constant), yielding

$$w(T') \propto \int_{-1}^1 dz \int_{-1}^1 dz' \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \frac{u(X^2 - T'^2)}{\sqrt{X^2 - T'^2}}. \tag{33}$$

We now introduce the three-dimensional unit vector \hat{u} with components $x = \sin \vartheta \cos \varphi$, $y = \sin \vartheta \sin \varphi$ and $z = \cos \vartheta$, and similarly the unit vector \hat{u}' defined with primed variables. The quantity X is the cosine of the angle ξ between \hat{u} and \hat{u}' , $X = \cos \xi(\hat{u}, \hat{u}')$. Thus (33) can be written as an integral over two solid angles,

$$w(T') \propto \iint \frac{u(\cos^2 \xi(\hat{u}, \hat{u}') - T'^2)}{\sqrt{\cos^2 \xi(\hat{u}, \hat{u}') - T'^2}} d\Omega d\Omega'. \tag{34}$$

Since the integrand depends only upon the relative angle between the two unit vectors \hat{u} and \hat{u}' , we can fix \hat{u}' along the z -axis and integrate over Ω . In this case $\xi = \vartheta$ and

$$w(T') \propto \int_{T'}^1 \frac{dz}{\sqrt{z^2 - T'^2}}. \tag{35}$$

The final result for $w(T)$, properly normalized, is

$$w(T) = \frac{1}{\pi} \ln \frac{1 + \sqrt{T(2 - T)}}{|1 - T|}. \tag{36}$$

From this distribution one finds explicitly $\langle T \rangle = 1$ and $\text{var } T = \frac{1}{6}$, consistent with the results of section 3 for $N = 2$.

4.3. The case $N = 2, \beta = 2$

The most general 2×2 unitary matrices $s^{(\pm)}$ can be written as

$$s^{(\pm)} = \begin{bmatrix} -\sqrt{\rho^\pm} e^{i(\alpha^\pm + \dot{\alpha}^\pm)} & \sqrt{\tau^\pm} e^{i(\alpha^\pm + \dot{\gamma}^\pm)} \\ \sqrt{\tau^\pm} e^{i(\gamma^\pm + \dot{\alpha}^\pm)} & \sqrt{\rho^\pm} e^{i(\gamma^\pm + \dot{\gamma}^\pm)} \end{bmatrix} \tag{37}$$

where $\alpha^\pm, \dot{\alpha}^\pm, \gamma^\pm, \dot{\gamma}^\pm$ are defined in $(0, 2\pi), 0 \leq \rho^\pm \leq 1, 0 \leq \tau^\pm \leq 1,$ and $\rho^\pm + \tau^\pm = 1.$ Note that if one takes $\dot{\alpha} = \alpha$ and $\dot{\gamma} = \gamma$ then equation (37) reduces to the $\beta = 1$ case (27). The invariant measure for $s^{(\pm)}$ is (Jalabert et al 1994, Baranger and Mello 1994)

$$d\mu^{(2)}(s^{(\pm)}) \propto d\tau^\pm d\alpha^\pm d\dot{\alpha}^\pm d\gamma^\pm d\dot{\gamma}^\pm. \tag{38}$$

In terms of these variables, the probability density $w(T')$ for $T' = 1 - T \in (-1, 1)$ is $w(T') \propto \int d\tau^+ d\tau^- d\alpha^+ d\dot{\alpha}^+ d\gamma^+ d\dot{\gamma}^+ d\alpha^- d\dot{\alpha}^- d\gamma^- d\dot{\gamma}^-$

$$\times \delta \left\{ T' - \frac{1}{2} \sqrt{\rho^+ \rho^-} [\cos(\alpha^+ - \alpha^- + \dot{\alpha}^+ - \dot{\alpha}^-) + \cos(\gamma^+ - \gamma^- + \dot{\gamma}^+ - \dot{\gamma}^-)] - \frac{1}{2} \sqrt{\tau^+ \tau^-} [\cos(\alpha^+ - \alpha^- + \dot{\gamma}^+ - \dot{\gamma}^-) + \cos(\gamma^+ - \gamma^- + \dot{\alpha}^+ - \dot{\alpha}^-)] \right\}. \tag{39}$$

The integrals over $\alpha^-, \dot{\alpha}^-, \gamma^-, \dot{\gamma}^-$ are trivial. Defining

$$\begin{aligned} \omega &= \frac{1}{2}(\alpha^+ + \dot{\gamma}^+ - \gamma^+ - \dot{\alpha}^+) \\ \psi &= \frac{1}{2}(\alpha^+ + \dot{\alpha}^+ - \gamma^+ - \dot{\gamma}^+) \\ \phi &= \frac{1}{2}(\alpha^+ + \dot{\alpha}^+ + \gamma^+ + \dot{\gamma}^+) \end{aligned} \tag{40}$$

we can write

$$w(T') \propto \int \delta \left[T' - \left(\sqrt{\tau^+ \tau^-} \cos \omega + \sqrt{\rho^+ \rho^-} \cos \psi \right) \cos \phi \right] d\tau^+ d\tau^- d\omega d\psi d\phi. \tag{41}$$

The integral over ϕ gives

$$w(T') \propto \int d\tau^+ d\tau^- d\psi d\omega \frac{u(Y^2 - T'^2)}{\sqrt{Y^2 - T'^2}} \tag{42}$$

where $u(Y^2 - T'^2)$ is a step function, and

$$Y = \sqrt{\tau^+ \tau^-} \cos \omega + \sqrt{\rho^+ \rho^-} \cos \psi. \tag{43}$$

We can write, in (42), $\omega = \varphi_1 - \varphi'_1, \psi = \varphi_2 - \varphi'_2$ and integrate over $\varphi_1, \varphi'_1, \varphi_2, \varphi'_2$ from 0 to 2π without altering the answer. We also write $\tau^+ = \cos^2 \theta, \tau^- = \cos^2 \theta'$ and get

$$w(T') \propto \int_0^{\pi/2} d\theta \sin \theta \cos \theta \int_0^{\pi/2} d\theta' \sin \theta' \cos \theta' \int \int \int \int_0^{2\pi} d\varphi_1 d\varphi'_1 d\varphi_2 d\varphi'_2 \frac{u(Y^2 - T'^2)}{\sqrt{Y^2 - T'^2}}. \tag{44}$$

Y can now be written as

$$Y = \cos \theta \cos \theta' \cos(\varphi_1 - \varphi'_1) + \sin \theta \sin \theta' \cos(\varphi_2 - \varphi'_2). \tag{45}$$

If we introduce the complex unit vector v

$$v = \begin{bmatrix} \cos \theta e^{i\varphi_1} \\ \sin \theta e^{i\varphi_2} \end{bmatrix} \tag{46}$$

with $0 \leq \theta \leq \pi/2, 0 \leq \varphi_{1,2} \leq 2\pi,$ and, similarly, the complex unit vector $v',$ defined with primed variables, we can express Y in terms of scalar products as

$$Y = \frac{1}{2}(v^\dagger v' + v'^\dagger v). \tag{47}$$

These scalar products are invariant with respect to unitary transformations. We also need to define a ‘solid angle’, invariant under the same operation. We first introduce the ‘arc element’

$$(ds)^2 = dv^\dagger dv = (d\theta)^2 + \cos^2 \theta (d\varphi_1)^2 + \sin^2 \theta (d\varphi_2)^2 \tag{48}$$

from which we extract a metric tensor g and construct the solid angle $d\Omega$ as

$$d\Omega = |\det g|^{1/2} d\theta d\varphi_1 d\varphi_2 = \sin \theta \cos \theta d\theta d\varphi_1 d\varphi_2. \tag{49}$$

Equation (44) for $w(T')$ now becomes

$$w(T') \propto \int \frac{u[\frac{1}{4}(v^\dagger v' + v'^\dagger v)^2 - T'^2]}{[\frac{1}{4}(v^\dagger v' + v'^\dagger v)^2 - T'^2]^{1/2}} d\Omega d\Omega'. \tag{50}$$

Just as in the case of the previous subsection we now notice that, if we fix v' in the integrand of equation (50) and integrate over Ω , we get a result independent of v' . The reason is that the scalar products in equation (50) are invariant under unitary transformations and we can always transform any given v' into a fixed vector: we choose this fixed vector as

$$v' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{51}$$

which implies $\theta' = \varphi_1' = 0$. We thus have

$$w(T') \propto \int \frac{u(\cos^2 \theta \cos^2 \varphi - T'^2)}{(\cos^2 \theta \cos^2 \varphi - T'^2)^{1/2}} \sin \theta \cos \theta d\theta d\varphi \tag{52}$$

where we have denoted φ_1 by φ . Doing the integral over θ we find

$$w(T') = \frac{1}{\pi^2} \int_0^{2\pi} \frac{\sqrt{\cos^2 \varphi - T'^2}}{\cos^2 \varphi} u(\cos^2 \varphi - T'^2) d\varphi. \tag{53}$$

The normalization constant in (53) was calculated by integrating over T' first and then over φ , since in both steps one finds elementary integrals. Performing the integrations in this order one can also verify that $\langle T \rangle = 1$ and $\text{var } T = \frac{1}{8}$, in agreement with the results of section 3 for $N = 2$.

The integral over φ in (53) leads to a hypergeometric function. First, doing the change of variables $\sin \varphi = \sqrt{1 - T'^2} \sin \theta$, we can write

$$w(T') = \frac{4}{\pi^2} (1 - T'^2) \int_0^{\pi/2} \frac{\cos^2 \theta d\theta}{[1 - (1 - T'^2) \sin^2 \theta]^{3/2}}. \tag{54}$$

Finally, we find (Gradshteyn and Ryzhik 1965, equation (3.681.1))

$$w(T) = \frac{1}{\pi} T(2 - T) F\left(\frac{1}{2}, \frac{3}{2}; 2; T(2 - T)\right). \tag{55}$$

As an algebraic check, one can verify that $\int w(T) dT = 1$ (Gradshteyn and Ryzhik 1965, equation (7.512.4)).

Acknowledgment

The authors are indebted to T H Seligman for a number of observations that led them to equations (6)–(8).

References

- Baranger H U and Mello P A 1994 *Phys. Rev. Lett.* **73** 142
—1995 *Phys. Rev. B* **51** 4703
—1996 unpublished
- Beenakker C W J and van Houten H 1991 *Solid State Physics* vol 44, ed H Ehrenreich and D Turnbull (New York: Academic) pp 1–228
- Blümel R and Smilansky U 1988 *Phys. Rev. Lett.* **60** 477
—1989 *Physica D* **36** 111
—1990 *Phys. Rev. Lett.* **64** 241
- Dyson F J 1962 *J. Math. Phys.* **3** 140
- Gradshteyn I S and Ryzhik I M 1965 *Table of Integrals, Series, and Products* (New York: Academic)
- Hua L K 1963 *Harmonic Analysis of Functions of Several Complex variables in the Classical Domain* (Providence, RI: AMS)
- Jalabert R A, Pichard J-L and Beenakker CWJ 1994 *Europhys. Lett.* **27** 255
- Lewenkopf C H and Weidenmüller H A 1991 *Ann. Phys., NY* **212** 53
- Mehta M L 1991 *Random Matrices* (New York: Academic)
- Mello P A and Seligman T H 1980 *Nucl. Phys. A* **344** 489
- Mello P A, Pereyra P and Seligman T H 1985 *Ann. Phys., NY* **161** 254
- Mello P A 1990 *J. Phys. A: Math. Gen.* **23** 4061
- Newton R G 1966 *Scattering Theory of Waves and Particles* (New York: McGraw-Hill)