

cylindrical $\nabla \times \mathbf{A} = \left[\frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial(sA_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right] \hat{\mathbf{z}}$ $\nabla f = \hat{\mathbf{s}} \frac{\partial f}{\partial s} + \hat{\phi} \frac{1}{s} \frac{\partial f}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial f}{\partial z}$

$$\nabla \cdot \mathbf{A} = \frac{1}{s} \frac{\partial(sA_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad d\tau = s ds d\phi dz \quad da = s d\phi dz \quad dl = ds \hat{\mathbf{s}} + s d\phi \hat{\phi} + dz \hat{\mathbf{z}}$$

spherical $\nabla = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$ $dl = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\phi \hat{\boldsymbol{\phi}}$

Polar (2d) $da = r dr d\theta$

$$\nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} \sin \theta A_\phi - \frac{\partial A_\theta}{\partial \phi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}}$$

$$da = r^2 d\theta \sin \theta d\phi \quad d\tau = dr da = r^2 dr d\theta \sin \theta d\phi$$

cartesian $dl = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$ $d\tau = dx dy dz$ $\nabla f = \hat{\mathbf{x}} \frac{\partial f}{\partial x} + \hat{\mathbf{y}} \frac{\partial f}{\partial y} + \hat{\mathbf{z}} \frac{\partial f}{\partial z}$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Gauss's Law: $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ $\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r^2}$ $V(r_b) - V(r_a) = - \int_{r_a}^{r_b} \mathbf{E} \cdot d\mathbf{l}$ $\nabla^2 V = 0 \Rightarrow \frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = 0$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{surface}} \frac{\sigma(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da' \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau'$$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad \mathbf{E} = -\nabla V \quad \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\zeta}}{\zeta^2} \rho(r') d\tau' \text{ where } \vec{\zeta} = \mathbf{r} - \mathbf{r}'$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int_{\text{volume}} d\tau' (r')^n P_n(\cos(\theta')) \rho(\mathbf{r}') \quad V_{\text{dipole}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \nabla \cdot \mathbf{D} = \rho_{\text{free}} \quad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad (\text{linear dielectrics}) \quad \mathbf{D} = \epsilon_0 (1 + \chi_e) \mathbf{E}$$

$$\mathbf{N} = \mathbf{p} \times \mathbf{E} \text{ or } \mathbf{N} = \mathbf{m} \times \mathbf{B}, \mathbf{F} = \nabla(\mathbf{p} \cdot \mathbf{E}) \text{ or } \nabla(\mathbf{m} \cdot \mathbf{B}) \quad \mathbf{p} = \int_{\text{volume}} \mathbf{r}' \rho(\mathbf{r}') d\tau' = \sum_i \mathbf{r}_i q_i \quad dq = \rho d\tau = \sigma da = \lambda dx$$

$$E_{\perp}^{\text{above}} - E_{\perp}^{\text{below}} = \frac{\sigma}{\epsilon_0} \quad W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad \nabla \times \mathbf{H} = \mathbf{J}_{\text{free}} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{d\mathbf{l}' \times \hat{\zeta}}{\zeta^2} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}' \times \hat{\zeta}}{\zeta^2} d\tau' \text{ where } \vec{\zeta} = \mathbf{r} - \mathbf{r}'$$

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

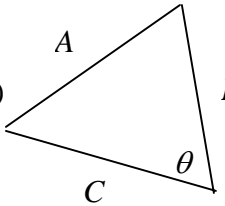
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \rightarrow \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad I = \lambda v \quad \mathbf{K} = \sigma \mathbf{v} \quad \mathbf{J} = \rho \mathbf{v} \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{n} \quad \mathbf{J}_b = \text{curl}(\mathbf{M})$$

$$\nabla \cdot \mathbf{B} = 0 \quad I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{a} \quad \mathbf{F} = Q\mathbf{v} \times \mathbf{B} \quad d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} = da \mathbf{K} \times \mathbf{B} = d\tau \mathbf{J} \times \mathbf{B}$$

$$\int_{\text{volume}} \nabla \cdot \mathbf{A} d\tau = \oint_{\text{surface}} \mathbf{A} \cdot d\mathbf{a} \quad \leftarrow \text{Divergence Theorem} \quad \text{Stokes Theorem} \rightarrow \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

$$\int_0^{\pi} d(\cos \theta) P_m(\cos \theta) P_l(\cos \theta) = \begin{cases} 0 & \text{if } m \neq l \\ \frac{2}{2m+1} & \text{if } m = l \end{cases}$$

$$A^2 = B^2 + C^2 - 2BC \cos(\theta)$$


$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \frac{1}{2}(3x^2 - 1), \quad \dots$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \quad \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \begin{cases} 0 & \text{if } n \neq m \\ a/2 & \text{if } n = m \end{cases}$$

$$\int \sin(kx) dx = -\frac{1}{k} \cos(kx) \quad \int \frac{\sin(\theta) d\theta}{\sqrt{1 - \cos(\theta)}} = 2\sqrt{1 - \cos(\theta)} \quad \int \cos(kx) dx = \frac{1}{k} \sin(kx)$$